Shielding Effectiveness of Composite Materials: Effect of Inclusion Shape

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The use of composite materials for electromagnetic shielding applications contributes to the effort of structure lightening in aerospace industry. In these materials the strong interaction between the electromagnetic field and the microstructure makes the standard numerical tools difficult to implement. Indeed these methods would involve an excessive number of degrees of freedom to describe details of the microstructure. An efficient way to overcome this problem is the use of homogenization techniques providing the effective properties of heterogeneous materials. These effective properties can then be introduced in standard numerical tools to estimate the behavior of shielding enclosures. A recent paper proposes an extension to microwave frequencies of quasistatic homogenization methods. It introduces a characteristic length for the microstructure in the case of a square array of circular 2-D conductive phases embedded in a dielectric matrix. In this paper, a method to identify this length parameter is proposed for random microstructures.

***Index Terms*—Electromagnetic compatibility, homogenization, Maxwell–Garnett model, phase distribution, 2-point probability function.**

I. Introduction

THE use of composite materials enables to combine advantages from different constituents. The focus in this paper is on epoxy resin filled in with carbon fibers, which provides lighter shielding structures compared to standard aluminum structures. In order to design composite structures, adequate modeling tools are required. Electromagnetic compatibility and especially shielding effectiveness (SE) of electronic device enclosures made of composite materials are affected by microstructural effects, and particularly by phase distribution and size effects. Although significant, these effects are rarely taken into account during the modeling stage due to the computation time involved when using standard numerical tools [1]–[4]. A solution is the use of a multiscale strategy such as homogenization.

Homogenization tools have been developed to estimate the behavior of composite materials [5]–[10], mostly under static conditions. However, when the wave frequency increases, the interaction between the electric field and the conductive phase of the composite becomes strong. Classical homogenization models such as Maxwell–Garnett model (MGM), cannot describe these dynamical effects. An extension of quasistatic homogenization methods to microwave frequencies has recently been proposed [11]. This model relies on the resolution of basic inclusion problems [12], [13] in which each phase of the composite is supposed to behave on average as a homogeneous ellipsoidal inclusion embedded in a homogeneous infinite medium (HIM). The shape of the inclusion (namely the axis ratio) is representative of the phase distribution in the real composite, so that different phase distributions can be considered. The extension to microwave frequencies is made through the introduction of a microstructure dependent characteristic length γ that is introduced in the definition of the HIM [11]. The main limitation is that this parameter γ has been identified for a square array of circular cylinders only. This paper intends to extend this homogenization model to more general cases of microstructures. It is shown that the use of 2-point probability functions (2PPF) provides the length parameter γ in the case of a random distribution of conductive ellipsoidal fibers in a dielectric matrix. The introduction of γ in the definition of the HIM allows—in addition to the shape of the ellipsoid in the inclusion problem—considering the effect of inclusion shape in the homogenization process. This effect is shown to be non negligible.

The first part of the paper presents the studied microstructures. Then the previously proposed dynamic homogenization model (DHM) to obtain the equivalent homogeneous medium (EHM) is briefly recalled. The proposed procedure to identify the characteristic length γ is then explained. In the results section, the shielding effectiveness of infinite sheets made of the EHM is compared to the SE of the corresponding composite material computed by the finite element method (FEM).

II. Microstructures

We consider three different microstructures (see Fig. 1) made of randomly disposed identical conductive inclusions (Phase 2, electric conductivity σ_2, dielectric permittivity ϵ_2, magnetic permeability μ_2 and volume fraction f_2) with circular (a), x-aligned (b) or y-aligned (c) elliptical inclusions embedded in a dielectric matrix (Phase 1, σ_1, ϵ_1, μ_1 and f_1). No contrast is considered on the magnetic permeability in this study ($\mu_1 = \mu_2$). These 2-D microstructures are equivalent to 3D microstructures composed of randomly disposed infinite conductive parallel fibers surrounded by a dielectric matrix.

III. Dynamic Homogenization Model (DHM)

The DHM provides the properties of the EHM to the composite material. The EHM is used to determine the SE of sheets made of biphasic composites. It can be used in a wider range of frequency than classical homogenization models [11].

Manuscript received November 09, 2012; revised January 15, 2013; accepted January 28, 2013. Date of current version May 07, 2013. Corresponding author: V. Préault (e-mail: valentin.preault@lgep.supelec.fr).

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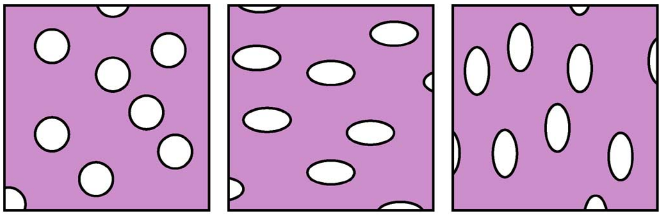
Digital Object Identifier 10.1109/TMAG.2013.2244865



(a) Microstructure a $a_x = 0.0707\ mm$, $a_y = 0.0707\ mm$, $f_2 = 19.63\ \%$

(b) Microstructure b $a_x = 0.1\ mm$, $a_y = 0.05\ mm$, $f_2 = 19.63\ \%$

(c) Microstructure c $a_x = 0.05\ mm$, $a_y = 0.1\ mm$, $f_2 = 19.63\ \%$

Fig. 1. Illustrations of the microstructures studied.

A. Effective Permittivity

The complex effective permittivity ($\epsilon^* = \epsilon - j(\sigma/\omega)$) of a biphasic composite made of isotropic constituents, is given by (1) [11]

$$\tilde{\epsilon}_u^* = \frac{\epsilon_1^* \frac{f_1}{\epsilon_\infty^* + N_u\left(\epsilon_1^* - \epsilon_\infty^*\right)} + \epsilon_2^* \frac{f_2}{\epsilon_\infty^* + N_u\left(\epsilon_2^* - \epsilon_\infty^*\right)}}{\frac{f_1}{\epsilon_\infty^* + N_u\left(\epsilon_1^* - \epsilon_\infty^*\right)} + \frac{f_2}{\epsilon_\infty^* + N_u\left(\epsilon_2^* - \epsilon_\infty^*\right)}} \tag{1}$$

where ϵ_∞^* is the complex permittivity of the infinite medium in the inclusion problems and N_u is the depolarization factor in the direction $\mathbf{u}$.

B. Depolarization Tensor

The depolarization tensor $\underline{\underline{N}}$ [5], [14] is computed from the ellipsoidal shape of the inclusion representing the phase in the inclusion problem. For dilute microstructures, made of identical inclusions, the shape of the representing inclusion can be taken as the shape of the inclusions in the real microstructure [5]. For the three microstructures studied, the corresponding shapes are infinite cylinders with a circular (a) or elliptical [(b) and (c)] cross-section. As shown in Appendix A, if the infinite medium is isotropic the depolarization tensor in the principal directions is given by

$$\underline{\underline{N}} = \begin{bmatrix} \frac{1}{\frac{a_x}{a_y}+1} & 0 & 0 \\ 0 & \frac{1}{\frac{a_y}{a_x}+1} & 0 \\ 0 & 0 & 0 \end{bmatrix}_{(x,y,z)} \tag{2}$$

where a_x and a_y are the elliptical cross-section semiaxes. The depolarization factor in the direction $\mathbf{u}$ can be obtained with $N_u =^t \mathbf{u}.\underline{\underline{N}}.\mathbf{u}$.

C. Infinite Medium

In the high frequency domain, the infinite medium is chosen as [11]

$$\epsilon_\infty^* = \epsilon_1^* + \epsilon_2^* \left(\frac{\gamma}{\lambda}\right)^2 \tag{3}$$

λ is the wavelength in the effective medium and γ is the characteristic size of the microstructure in the incident wave direction. For low frequencies (high λ), the model becomes equivalent to MGM. In the case of a square array of parallel cylindrical fibers embedded in a dielectric matrix, as studied in [11], γ is the diameter ($\varnothing$) of the fibers. But γ is unknown for random microstructures. In the next section, it is shown that it can be identified from 2PPF.

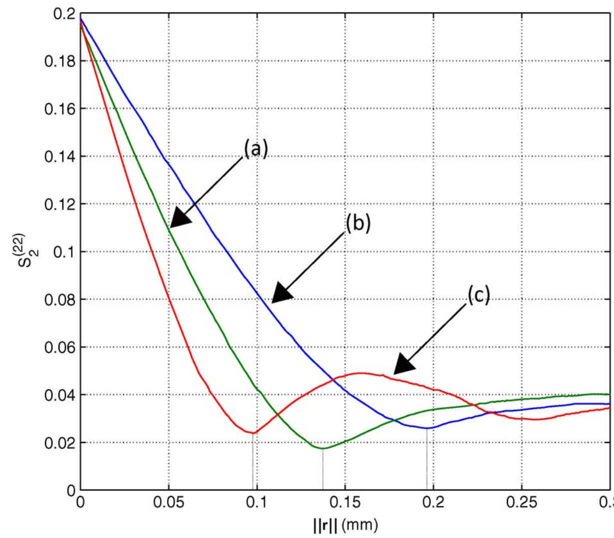


Fig. 2. 2-point probability functions of the conductive phase (Phase 2) for the three microstructures in the horizontal (incident wave) direction. Computations have been processed on a 2 mm side square filled with 50 inclusions.

IV. Characteristic Size of Random Microstructures

2PPF provide relevant information on microstructures and are often used to characterize composite materials [15]. They can be expressed, for the phase i as

$$S_2^{(ii)}(\mathbf{r}) = \left\langle \mathcal{I}^{(i)}(\mathbf{x})\mathcal{I}^{(i)}(\mathbf{x}+\mathbf{r}) \right\rangle \tag{4}$$

where $\mathcal{I}^{(i)}(\mathbf{x})$ is the *indicator function* ($\mathcal{I}^{(i)}(\mathbf{x}) = 1$ if $\mathbf{x} \in \mathrm{Phase}\ i$ and $\mathcal{I}^{(i)}(\mathbf{x}) = 0$ otherwise) and $\langle . \rangle$ is the volume averaging operator. According to our investigations, the 2PPF of the conductive phase (Phase 2) provide the characteristic length γ of the microstructure. This characteristic size is the length for which $S_2^{(22)}$ is minimum in the direction parallel to the wave vector $\mathbf{k}$

$$\gamma = \|\mathbf{r}_m\| \text{ with } S_2^{(22)}(\mathbf{r}_m) = \min\left(S_2^{(22)}(\mathbf{r})\right), \quad \mathbf{r}//\mathbf{k}. \tag{5}$$

The 2PPF are plotted on Fig. 2. These curves have been numerically built from a random selection of $\mathbf{x}$ and an exploration of the microstructure at $\mathbf{x}+\mathbf{r}$.

The characteristic sizes of the microstructures can be read from these curves: $\gamma_a = 0.1414$ mm, $\gamma_b = 0.1975$ mm and $\gamma_c = 0.09825$ mm. The property $S_2^{(22)}(0) = f_2$ is retrieved. It can be noticed that $\gamma_a = 2a_x = \varnothing$, which is consistent with the choice in [11] for identical circular fibers.

V. Modeling Results

The SE of infinite homogeneous sheets made of the EHM obtained with the DHM and MGM can be computed analytically. It has been shown that, in the case of homogeneous sheets, the FEM computation gives a SE estimate equal to the analytical solution. [11]. The analytical results are compared to the SE computed by FEM using the software $\mathrm{COMSOL_{TM}}$ on meshes of the corresponding microstructures (see Figs. 4–6).

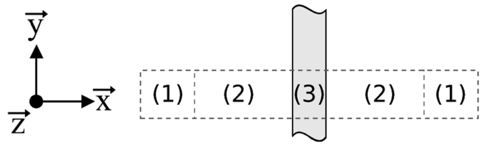


Fig. 3. Scheme of SE calculations configurations with the FEM domain (dashed grey): PML (1), Air (2), and Studied sheet (3).

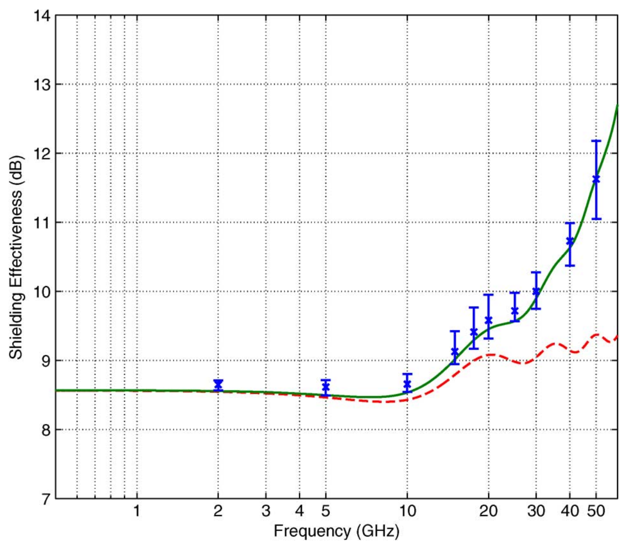


Fig. 4. Shielding effectiveness of a 6 mm sheet made of microstructure (a), FEM (crosses with error bars), MGM (dashed line), and DHM (line) results.

The SE has been evaluated on 6 mm thick sheets surrounded by air. Constituents properties are $\sigma_1 = 1\ \mathrm{S.m}^{-1}$, $\sigma_2 = 1000\ \mathrm{S.m}^{-1}$, $\epsilon_1 = 2\epsilon_0$, $\epsilon_2 = \epsilon_0$, $\mu_1 = \mu_2 = \mu_0$. The incident field is a monochromatic perpendicular plane wave propagating along direction $\mathbf{x}$ with the electric field along $\mathbf{y}$ (see Fig. 3).

Due to computational resource limitation, the domain modeled by FEM is limited to a heigth of 0.8 mm (direction $\mathbf{y}$) and is surrounded by two perfectly matched layers (PML). Periodic conditions are applied on the top and the bottom of the domain.

In Figs. 4–6, the crosses are obtained from the average values of the SE on a set of 20 FEM calculations with similar properties (volume fraction, fiber shape, material properties) but different (random) practical realizations. The error bars have been generated from the extreme values obtained by FEM. Their size is directly connected to the size of the modeled domain. The domain used here does not include enough fibers to be fully representative of the microstructure. This effect is dependant of the fiber orientations, inducing a larger variation for (c) and a lower one for (b) than for (a). Bigger modeling domains would reduce error bars at the price of an increased computational cost.

Standard MGM fails to predict the SE at high frequency. The DHM provides an estimate of SE close to FEM calculations. Each microstructure feature is well captured with the DHM at high frequency. As explained in [11], it is expected that the error of the DHM becomes significant when the frequency is increased so that the skin-depth becomes of the order of magnitude of the characteristic size γ.

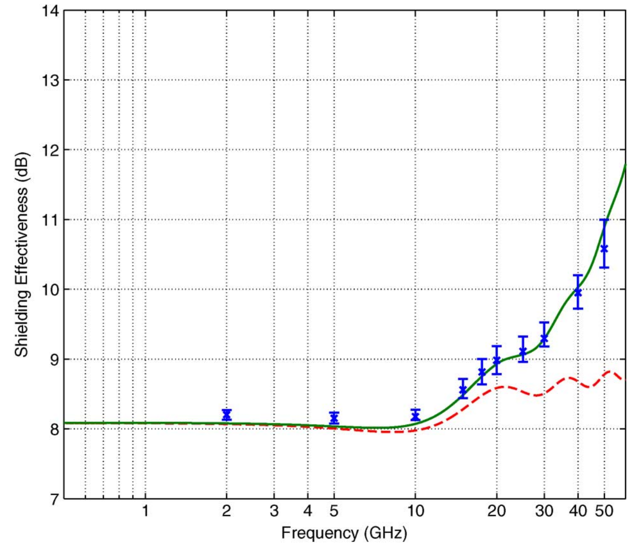


Fig. 5. Shielding effectiveness of a 6 mm sheet made of microstructure (b), FEM (crosses with error bars), MGM (dashed line), and DHM (line) results.

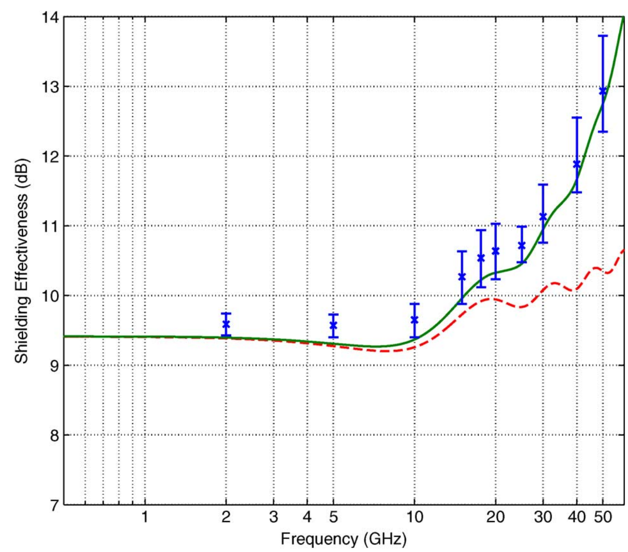


Fig. 6. Shielding effectiveness of a 6 mm sheet made of microstructure (c), FEM (crosses with error bars), MGM (dashed line), and DHM (line) results.

The extension of quasistatic homogenization models to microwave frequencies can be used for a large range of random microstructures. Microstructures similar to (b) and (c) have been modeled. They are composed of aligned elliptical inclusions but oriented in other directions than $\mathbf{x}$ or $\mathbf{y}$. The results presented in Fig. 7 for various orientations show that inclusion shape has a significant effect on the SE properties of composite materials. This effect is captured through the phase distribution and the characteristic size γ of the microstructure.

Other microstructures, such as spherical or ellipsoidal conductive inclusions surrounded by a dielectric matrix can be modeled. This is part of the work in progress.

VI. Conclusion

A dynamic homogenization model for the effective permittivity of composite materials has been used. This model is based

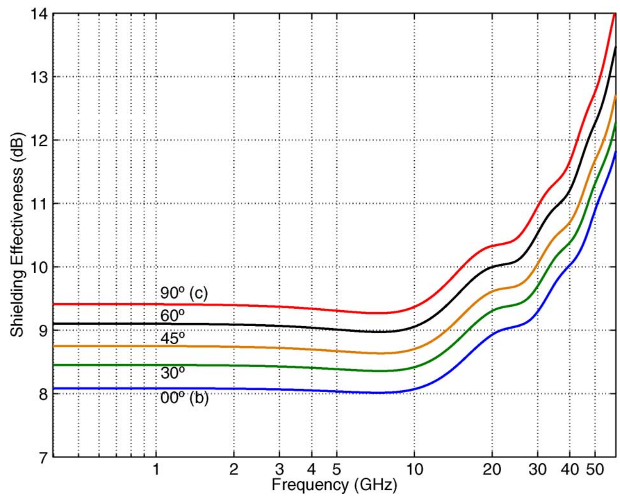


Fig. 7. Shielding effectiveness of various composite sheets computed with the DHM.

on the introduction of a characteristic length in quasistatic homogenization approaches. This model was initially restricted to a square array of circular cylinders. It is shown in this paper that the characteristic length can be identified on random microstructures from the computation of 2-point probability functions. Together with phase distribution, this characteristic length allows the definition of the properties of the equivalent homogeneous medium. This method is similar to the Maxwell-Garnett estimate for low frequencies but extends its domain of validity at high frequencies. The method is implemented to define the shielding effectiveness of composite materials with very low computational cost. The results have been satisfactorily compared to finite element computations. Such approaches make possible the design of composite shielding enclosures for electromagnetic compatibility applications.

Appendix

A. Depolarization Tensor

Stratton [14] gives the depolarization factors N_j of an ellipsoidal inclusion embedded in an infinite isotropic medium (the ellipsoid semiaxes are aligned with the basis axes)

$$N_j = \frac{a_x a_y a_z}{2} \int_0^\infty \frac{1}{\left(a_j^2+s\right)\sqrt{\left(a_x^2+s\right)\left(a_y^2+s\right)\left(a_z^2+s\right)}} ds \tag{6}$$

where $j=\{x,y,z\}$ and the a_j are the ellipsoid semiaxes.

In our case, the inclusion is an infinite cylinder with an elliptical cross-section. Since the semiaxis a_z is equal to ∞, the depolarization factor in the direction $j=\{x,y\}$ can be written

$$N_j = \frac{a_x a_y}{2} \int_0^\infty \frac{1}{\left(a_j^2+s\right)\sqrt{\left(a_x^2+s\right)\left(a_y^2+s\right)}} ds. \tag{7}$$

The depolarization tensor is then equal to

$$\underline{\underline{N}} = \begin{bmatrix} \frac{1}{\frac{a_x}{a_y}+1} & 0 & 0 \\ 0 & \frac{1}{\frac{a_y}{a_x}+1} & 0 \\ 0 & 0 & 0 \end{bmatrix}_{x,y,z}. \tag{8}$$

Acknowledgment

This work was supported by the Project FUI-AAP10 SYRENA.

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