Influence of Mechanical Boundary Conditions on Magnetoelectric Sensors

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Magnetic field sensors are an important application for magnetoelectric composite materials. In these devices the external magnetic field is converted into an electric voltage. The sensitivity of the sensor is known to depend on different factors, including geometrical and material parameters. This work deals with the modeling of the influence of mechanical boundary conditions on the sensitivity of magnetoelectric sensors.

***Index Terms*—Finite element formulation, frequency effect, magnetoelectric effect, magnetostriction, piezoelectricity.**

I. Introduction

MAGNETOELECTRIC (ME) composites meet an increasing interest in materials science research. A main application using magnetoelectric composites is magnetic field sensing. Magnetoelectric sensors mostly consist in layered piezoelectric (PZ) and magnetostrictive (MM) materials. A key point in ME sensing is to superimpose a harmonic magnetic field at structure resonance frequency to the static—bias—magnetic field to be measured [1]–[3]. Due to the nonlinear behavior of magnetostriction, the obtained harmonic electric voltage depends on these two signals [4]. Many factors can modify the sensitivity of the sensor and particularly material parameters. Wu *et al.* [5] have experimentally demonstrated that the sensitivity of the sensor is also influenced by the mechanical loadings. Recently, Biju *et al.* [6] investigate the influence of different mechanical boundary conditions on the electric response without considering the nonlinear magneto-elastic behavior. This paper intends to investigate the influence of the mechanical boundary conditions on ME sensor sensitivity. It is based on a finite element model for ME effect under simultaneous harmonic and static excitations [4], [7]. Here, an additional term in the coupled constitutive law is introduced to consider the impact of an applied stress. It is shown that the stress significantly modifies the sensor sensitivity through its effects on both the static and dynamic behavior.

II. ME Sensor Configuration

Fig. 1 shows the ME sensor configuration. This ME sensor is a trilayer subjected to a uniform pressure on its right edge. The left edge of the structure is clamped. A static magnetic field $\mathbf{H}_{dc}$ and a harmonic magnetic field $\mathbf{h}_{ac}$ are applied simultaneously along x-direction. Under static mechanical loadings, the magnetic behavior of the magnetostrictive materials is modified. In accordance with standard experimental configurations, the amplitude of $\mathbf{h}_{ac}$ is much smaller than the amplitude of $\mathbf{H}_{dc}$. The PZT is used for the middle layer, its behavior obeys a standard linear constitutive law. Terfenol-D is used for the inner and outer layers, its behavior is nonlinear. The sensor is 5 mm long with layer depths equal to 50 μm for the MM and 200 μm for the PZ. The working plane is $x-y$ plane.

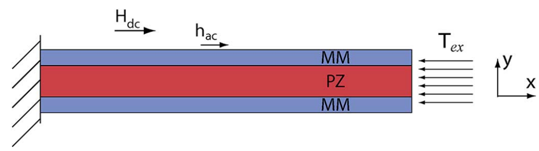

Fig. 1. ME sensor configuration: trilayer structure of magnetostrictive (MM) and piezoelectric (PZ) materials subjected to an harmonic magnetic excitation hac superimposed to a static magnetic field $\mathbf{H}_{dc}$. The mechanical boundary condition is a uniaxial stress $\mathbf{T}_{ex}$ applied on the right side of the sensor, the left side being clamped.

We denote by $\mathbf{T}$ the stress tensor, $\mathbf{f}$ the driving force, $\mathbf{u}$ the displacement, $\mathbf{S}$ the strain tensor, $\mathbf{E}$ the electric field, $\mathbf{D}$ the electric induction and $\mathbf{M}$ the magnetization. We note $\widetilde{X}(\widetilde{a},\widetilde{b})$ the small variation of a function X, depending on the variable a and b, around a polarization point $X_0(a_0,b_0)$

$$X = X_0 + \widetilde{X} \qquad \widetilde{X} = \frac{\partial X}{\partial a}(a_0,b_0)\widetilde{a} + \frac{\partial X}{\partial b}(a_0,b_0)\widetilde{b}. \quad (1)$$

III. Equilibrium Equations

The mechanical equilibrium is given by

$$\mathrm{div}\mathbf{T} + \mathbf{f} = \rho_m \frac{\partial^2 \mathbf{u}}{\partial t^2} \quad (2)$$

where ρ_m is the mass density.

No dielectric induction $\mathbf{D}$, electric current $\mathbf{J}$ nor charge density ρ are considered in this paper. The electromagnetic equilibrium is then given by

$$\mathbf{curlH} = \mathbf{0} \quad (3)$$

$$\mathrm{div}\mathbf{D} = 0. \quad (4)$$

IV. Constitutive Laws

An applied static magnetic field $\mathbf{H}_{dc}$ imposes a polarization point for the ME sensor. The additional harmonic magnetic field $\mathbf{h}_{ac}$ introduces a small variation around the polarization point given by the static field $\mathbf{H}_{dc}$. In this part, we first introduce the

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constitutive laws written in a general form for the determination of the polarization point as a function of $\mathbf{H}_{dc}$. To describe the behavior of materials at a given polarization point, we then write the constitutive laws in linearized form. Contrarily to previous works [4], [7], these constitutive laws include an additional term that cannot be neglected so as to consider the impact of an applied static stress.

A. Piezoelectric Behavior

Linear behavior is considered for the piezoelectric material

$$\begin{pmatrix} \widetilde{\mathbf{T}} \\ \widetilde{\mathbf{D}} \end{pmatrix} = \begin{pmatrix} \mathbf{c}_{pz} & -\mathbf{e}_{pz}^t \\ \mathbf{e}_{pz} & \boldsymbol{\epsilon}_{pz} \end{pmatrix} \begin{pmatrix} \widetilde{\mathbf{S}} \\ \widetilde{\mathbf{E}} \end{pmatrix} \tag{5}$$

where $\mathbf{c}_{pz}$ is the stiffness tensor at constant electric field, $\boldsymbol{\epsilon}_{pz}$ the electric permittivity at constant strain and $\mathbf{e}_{pz}$ the piezoelectric coefficients.

B. Magneto-Elastic Behavior

1) General Form: The total strain $\mathbf{S}$ is divided into the elastic strain $\mathbf{S}^e$ and the magnetostriction strain $\mathbf{S}^\mu$, $\mathbf{S} = \mathbf{S}^e + \mathbf{S}^\mu$ [8]. The magnetostrictive constitutive law is written according to the following assumptions[4]:

- magnetostriction strain is assumed isochoric and isotropic;
- magnetic induction and magnetization are assumed to be parallel;
- magnetostriction strain is modeled by a quadratic function of the magnetization.

The magnetostriction strain can then be written

$$s_{ij}^\mu = \frac{\beta}{2}\left(3b_ib_j - \delta_{ij}\|\mathbf{B}\|\right)\frac{\|\mathbf{M}\|^2}{\|\mathbf{B}\|^2} \tag{6}$$

where β is a magnetostrictive coefficient identified from experimental magnetostriction curves, δ the Kronecker symbol ($\delta_{ij} = 1$ if and only if $i = j$), $\|\mathbf{M}\|$ and $\|\mathbf{B}\|$ the norm of $\mathbf{M}$ and $\mathbf{B}$ respectively.

Magnetic behavior (without considering stress) is described by a Langevin function

$$M = M_s\left(\frac{1}{\tanh(\alpha H)} - \frac{1}{\alpha H}\right) \tag{7}$$

with M_s the saturation magnetization. The constant α can be defined as $\alpha = 3\chi_0/M_s$ with χ_0 the initial susceptibility of the anhysteretic magnetization curve [9].

The magnetoelastic constitutive laws are written [7], [10]–[12]

$$\begin{aligned} t_{ij} &= C_{ijkl}^{ms}s_{kl} - t_{kl}^\mu \\ h_i &= \nu_{ij}b_j - \frac{\partial t_{kl}^\mu}{\partial b_i}\left(s_{kl} - s_{kl}^\mu\right) \end{aligned} \tag{8}$$

where C_{ijkl}^{ms} is the stiffness tensor of the magnetostrictive material, ν_{ij} its reluctivity, $t_{kl}^\mu = C_{ijkl}^{ms}s_{kl}^\mu$. In the case of isochoric magnetostriction, we have $s_{kk}^\mu = 0$, the relationship between t_{kl}^μ and s_{kl}^μ can be simplified using the shear modulus μ^*

$$t_{kl}^\mu = 2\mu^* s_{kl}^\mu. \tag{9}$$

If there is no mechanical loadings, $s_{kl} - s_{kl}^\mu = 0$, the second equation of (8) is simplified into: $h_i = \nu_{ij}b_j$. In the case of an applied stress, $s_{kl} - s_{kl}^\mu \neq 0$, and this term allows directly the introduction of the stress in the "static" magnetic constitutive law (8).

2) Linearized Form: In order to investigate the magneto-elastic behavior under harmonic excitation, we introduce the "small signal" magneto-elastic constitutive laws. Considering $\mathbf{B}$ and $\mathbf{S}$ as state variables, the "small signal" magnetoelastic constitutive laws are obtained by calculating the differential of (8)

$$\widetilde{t}_{ij} = C_{ijkl}^{ms}\widetilde{s}_{kl} - \frac{\partial t_{ij}^\mu}{\partial b_k}\widetilde{b}_k \tag{10}$$

$$\widetilde{h}_i = -\frac{\partial t_{kl}^\mu}{\partial b_i}\widetilde{s}_{kl} + \left[\nu_{ij} - \frac{\partial^2 t_{kl}^\mu}{\partial b_i \partial b_j}\left(s_{kl} - s_{kl}^\mu\right) + \frac{\partial t_{kl}^\mu}{\partial b_i}\frac{\partial s_{kl}^\mu}{\partial b_j}\right]\widetilde{b}_j. \tag{11}$$

From (9), the term $\partial t_{kl}^\mu/\partial b_i$ is calculated as previously detailed in [7].

Equation (11) introduces the term of equivalent reluctivity $\widetilde{\nu}$

$$\widetilde{\nu}_{ij} = \left[\nu_{ij} - \frac{\partial^2 t_{kl}^\mu}{\partial b_i \partial b_j}\left(s_{kl} - s_{kl}^\mu\right) + \frac{\partial t_{kl}^\mu}{\partial b_i}\frac{\partial s_{kl}^\mu}{\partial b_j}\right]. \tag{12}$$

This equation shows that, as in the "static" case, the application of a stress has to be considered with the term $(s_{kl} - s_{kl}^\mu)$. Contrarily to previous works [4], [7], the term $\partial^2 t_{kl}^\mu/\partial b_i \partial b_j$ has to be evaluated, with an additional derivation of the magnetostriction strain. The applied stress is then considered twice: first for the determination of the nonlinear magnetization curve for the static excitation, the permeability being influenced by the stress as detailed with (8), and secondly in the additional term of the equivalent reluctivity $\widetilde{\nu}$ for the harmonic excitation.

In our case, at a given polarization point, depending on static magnetic field $\mathbf{H}_{dc}$ and mechanical loadings, the constitutive laws of the magnetostrictive material can then be written in the matrix form

$$\begin{pmatrix} \widetilde{\mathbf{T}} \\ \widetilde{\mathbf{H}} \end{pmatrix} = \begin{pmatrix} \widetilde{\mathbf{c}}_{ms} & -\widetilde{\mathbf{q}}_{ms}^{\mathrm{t}} \\ -\widetilde{\mathbf{q}}_{ms} & \widetilde{\nu}_{ms} \end{pmatrix} \begin{pmatrix} \widetilde{\mathbf{S}} \\ \widetilde{\mathbf{B}} \end{pmatrix}. \tag{13}$$

At a polarization point, the overall constitutive laws of magnetoelectric composite materials are combined from (5) and (13) and written in the following system:

$$\begin{pmatrix} \widetilde{\mathbf{T}} \\ \widetilde{\mathbf{D}} \\ \widetilde{\mathbf{H}} \end{pmatrix} = \begin{pmatrix} \widetilde{\mathbf{c}} & -\widetilde{\mathbf{e}}^{\mathrm{t}} & -\widetilde{\mathbf{q}}^{\mathrm{t}} \\ -\widetilde{\mathbf{e}} & \widetilde{\boldsymbol{\epsilon}} & \mathbf{0} \\ -\widetilde{\mathbf{q}} & \mathbf{0} & \widetilde{\boldsymbol{\nu}} \end{pmatrix} \begin{pmatrix} \widetilde{\mathbf{S}} \\ \widetilde{\mathbf{E}} \\ \widetilde{\mathbf{B}} \end{pmatrix} \tag{14}$$

with $\widetilde{\mathbf{e}} = \mathbf{0}$ for the magnetostrictive material and $\widetilde{\mathbf{q}} = \mathbf{0}$ for the piezoelectric material.

V. Finite Element Formulation

In this part, the 2-D finite element formulation is developed using Galerkin method and nodal element discretization. The modeling of the magneto-electric sensor is done in two steps. The first part uses a static finite element formulation in order to calculate the magnetic vector potential, and then the magnetic induction and magnetization in all elements of the geometry discretization. These values depend on the applied static magnetic field and mechanical loadings. This first part is then used to estimate the parameters of the constitutive laws of the magnetostrictive material, as presented in (10) and (11). These parameters

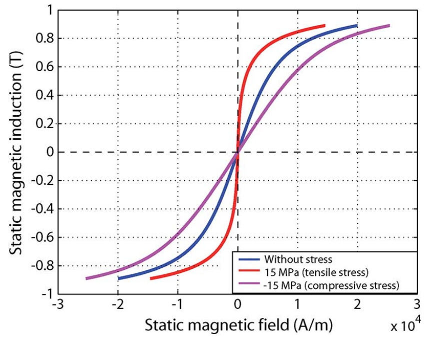

Fig. 2. Magnetic induction curves in the absence of stress and under uniaxial compressive and tensile stress (magnitude 15 MPa).

are implemented in the second part using a harmonic finite element formulation. Finally, electric voltage V is obtained in the second part at resonance frequency.

A. Static Formulation

In this part only the magnetic problem is taken into consideration. The magnetic constitutive law reads

$$h_i = \nu_{ij} b_j - \frac{\partial t^{\mu}_{kl}}{\partial b_i}\left(s_{kl} - s^{\mu}_{kl}\right) \tag{15}$$

where $s_{kl} - s^{\mu}_{kl}$ is used in order to introduce the stress applied on the sensor. $\partial t^{\mu}_{kl}/\partial b_i$ is a function of the magnetic induction $\mathbf{B}$ and magnetization $\mathbf{M}$. Magnetic induction $\mathbf{B}$ is assumed to be in the working plane (x, y) and invariant with z. Therefore magnetic vector potential, $\mathbf{B} = \mathbf{curl}(\mathbf{a})$, is along z-direction and invariant with z.

The finite element formulation is deduced from magnetic equilibrium (3)

$$[\mathbb{K}_{aa}]\mathbf{a} = \mathbf{0} \tag{16}$$

with $\mathbb{K}_{aa}$ the magnetic stiffness matrix. The Dirichlet boundary conditions are related to the applied static magnetic field $\mathbf{H}_{dc}$, corresponding to the magnetic field measured by the sensor. Due to the nonlinearity of the problem, (16) is solved according to fixed point technique.

In the case considered here, the applied stress $\mathbf{T}_{ex}$ is a uniform uniaxial stress along x-direction. $\mathbf{S} - \mathbf{S}^{\mu}$ is obtained using the Hooke law: $(\mathbf{S} - \mathbf{S}^{\mu}) = \mathbf{c}^{-1} : \mathbf{T}_{ex}$.

Fig. 2 presents the magnetization curves for tensile and compressive stress under static magnetic field.

As expected for materials with positive magnetostriction, the permeability increases under tension, and decreases under compression. As a consequence, the magnetic induction is higher in the magnetostrictive parts of the sensor if a tensile stress is applied.

B. Harmonic Problem

As the harmonic magnetic field $\mathbf{h}_{ac}$ makes a variation around a polarization point given by the static field $\mathbf{H}_{dc}$, we need to use the linearized form of the constitutive laws for the magnetostrictive material. The finite element formulation is a coupled formulation between mechanical, magnetic and electrical problems. From mechanical equilibrium (2) and constitutive laws (14), the finite element formulation is

$$\int_{\Omega} [N]\left(\mathrm{div}(\mathbf{cS} - \mathbf{eE} - \mathbf{qB}) + \mathbf{f} - \rho_m \frac{\partial^2 \mathbf{u}}{\partial t^2}\right) d\Omega = 0 \tag{17}$$

where Ω corresponds to the study domain, Γ_s the boundaries of the study domain, and N the 1st order triangular shape functions. Using the same development as in previous works [4], [7], (17) becomes

$$\begin{aligned} &\oint_{\Gamma_s}[N]\mathbf{cS}d\Omega - \int_{\Omega}\mathbf{Scgrad}[N]d\Omega - \oint_{\Gamma_s}[N]\mathbf{eE}d\Omega \\ &+ \int_{\Omega}\mathbf{Eegrad}[N]d\Omega - \oint_{\Gamma_s}[N]\mathbf{qB}d\Omega + \int_{\Omega}\mathbf{Bqgrad}[N]d\Omega \\ &+ \rho_m\omega^2\int_{\Omega}[N]\mathbf{u}d\Omega = -\int_{\Omega}[N]\mathbf{f}d\Omega. \end{aligned} \tag{18}$$

With Dirichlet boundary conditions, using $\mathbf{B} = \mathbf{curla}$ and $\mathbf{E} = -\mathbf{grad}V$ (18) is simplified into

$$\begin{aligned} &-\int_{\Omega}\mathcal{D}\mathbf{ucgrad}[N]d\Omega + \int_{\Omega}\mathbf{grad}V\mathbf{egrad}[N]d\Omega \\ &+ \int_{\Omega}\mathbf{curlaqgrad}[N]d\Omega \\ &+ \rho_m\omega^2\int_{\Omega}[N]\mathbf{u}d\Omega = -\int_{\Omega}[N]\mathbf{f}d\Omega. \end{aligned} \tag{19}$$

Equation (19) can be written in the matrix form

$$(\mathbb{K}_{uu} - \omega^2\mathbb{M})\mathbf{u} + \mathbb{K}_{up}V + \mathbb{K}_{ua}\mathbf{a} = \mathbf{f} \tag{20}$$

$$\text{with} \quad \begin{cases} \mathbb{K}_{uu} = \sum_e \int_{\Omega^e} \mathcal{D}[N]\mathbf{c}\mathcal{D}[N]d\Omega^e \\ \mathbb{K}_{up} = -\sum_e \int_{\Omega^e} \mathbf{grad}[N]\mathbf{egrad}[N]d\Omega^e \\ \mathbb{K}_{ua} = -\sum_e \int_{\Omega^e} \mathbf{grad}[N]\mathbf{qgrad}[N]d\Omega^e \end{cases} \tag{21}$$

Ω^e is the partial domain. Equation (20) can be complemented with a damping term $\mathbb{D}(\partial\mathbf{u}/\partial t) = j\omega\alpha\mathbb{K}_{uu}\mathbf{u}$ with α the damping coefficient. Noting $\mathbb{K}^*_{uu} = \mathbb{K}_{uu} + j\omega\alpha\mathbb{K}_{uu} - \omega^2\mathbb{M}$ gives

$$\mathbb{K}^*_{uu}\mathbf{u} + \mathbb{K}_{up}V + \mathbb{K}_{ua}\mathbf{a} = \mathbf{f}. \tag{22}$$

The electromagnetic finite element formulation is established in a similar way, detailed in previous work [7]. The electromagnetic formulation uses the potential vector magnetic $\mathbf{a}$ and potential scalar electric V. The overall finite element system is finally described by the following system:

$$\begin{pmatrix} \mathbb{K}_{uu} & \mathbb{K}_{up} & \mathbb{K}_{ua} \\ \mathbb{K}_{pu} & \mathbb{K}_{pp} & \mathbb{O} \\ \mathbb{K}_{au} & \mathbb{O} & \mathbb{K}_{aa} \end{pmatrix} \begin{pmatrix} \mathbf{u} \\ V \\ \mathbf{a} \end{pmatrix} = \begin{pmatrix} \mathbf{f} \\ 0 \\ \mathbf{0} \end{pmatrix}. \tag{23}$$

C. Modeling Procedure

The modeling procedure is summarized in Fig. 3. After the determination of the constitutive law (13) from the static problem, the first part of the harmonic step is to determine the first mechanical resonance frequency of the structure, corresponding to the working frequency of $\mathbf{h}_{ac}$ for the harmonic

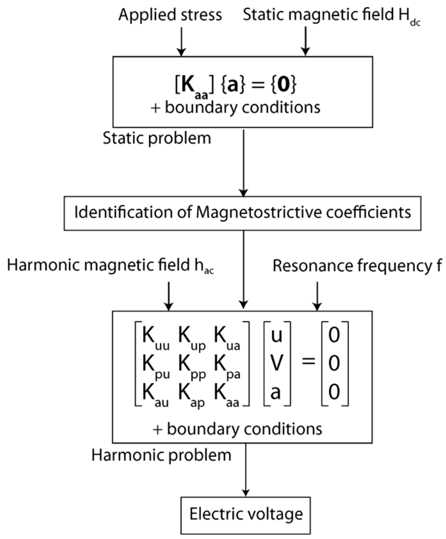

Fig. 3. Modeling procedure for the ME sensor, the mechanical boundary conditions are considered in both static and harmonic problems.

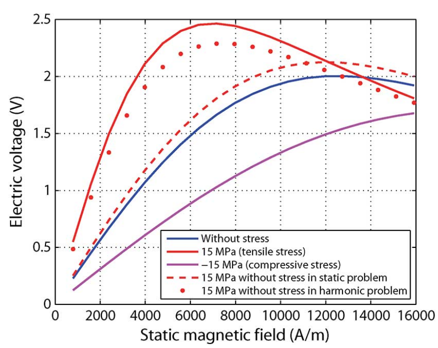

Fig. 4. Influence of the applied stress on the ME effect, according to different approximations.

problem. The mechanical boundary conditions in harmonic problem is taken into account by considering the complete equivalent reluctivity term (12). In this context, considering a given polarization point, the electric voltage is calculated for different frequencies until reaching the first maximum. The corresponding frequency is then kept constant for the harmonic step.

VI. Results and Discussion

For the considered sensor, the first resonance frequency has been found to be about 160 kHz. As the structure is a trilayer, this first resonance frequency corresponds to a longitudinal mode, due to the horizontal symmetry of the system. We then investigate the influence of the mechanical loadings in the harmonic case. Fig. 4 shows the electric voltage as a function of static magnetic field for different approximations: first without mechanical loading, second considering tensile or compressive stress in both static and harmonic problems, third without considering tensile stress in the static problem (only in the harmonic problem), and finally without considering stress in the harmonic problem (12).

Considering the no-stress curve, the electric voltage first increases as a function of the static magnetic field and then decreases towards 0, as already shown in previous works [4], [7]. The linear region of the initial—increasing—stage is usually the operating range for these sensors. Fig. 4 also shows how the applied stress modifies the sensitivity of the ME sensor: tension increases the sensitivity of the sensor whereas compression decreases it, as observed experimentally in [5]. The main reason for this evolution is that tension increases the permeability and thus the magnitude of the magnetostriction strain. As a consequence the electric voltage is higher for a given external magnetic field. Compression has an opposite effect. The numerical analysis performed shows that the static contribution of the effect of stress on the sensitivity is greater than the dynamic contribution. The proposed model can be used as a tool for structural optimisation of magneto-electric sensors under combined magneto-mechanical loadings.

VII. Conclusion

This paper proposes an improvement of a ME model previously published in order to take into account mechanical loadings in the constitutive law of magnetostrictive materials. It is shown that the stress has an impact not only on the magnetization curve, but also on the equivalent reluctivity of the harmonic problem. The model is in accordance with experimental results from the literature. It allows to dissociate the different contributions of the stress and gives an insight into the possible strategies for sensor optimization. It could also be used to calibrate the packaging constraints in commercial magneto-electric sensors.

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