The Role of the Out of Plane Component of E and H in 2D Computation of Extrinsic Magneto-Electric Problem Using E-H Formulation

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This paper deals with the modeling of the magnetoelectric effect in composite materials. This work focuses on the modeling of a magnetoelectric multilayer using a full 2D finite element model. It is shown that the magnetic induction in the direction normal to the plane can be neglected in 2D formulations under low frequency. An application is made on a bilayer and on an asymmetric multilayer magnetic sensor based on extrinsic magnetoelectric effect. Both structures give approximately the same first resonance frequency but a higher electric voltage is obtained in the asymmetric multilayer.

***Index Terms*—Finite element formulation, frequency effect, magnetoelectric effect, magnetostriction, piezoelectricity.**

I. Introduction

Magnetoelectric (ME) phenomenon is the existence of a magnetization induced by an electric polarization, or conversely an electric polarization induced by a magnetization. In this paper we focus on the modeling of magneto-electric laminate composites using two-dimensional finite element method. The magneto-electric laminate composite consists in layered piezoelectric and magnetostrictive materials that produce the ME effect through an elastic interaction. Previous papers have described different formulations for magnetoelectric effect for two dimensional problems: The use of electric and magnetic scalar potentials has been proposed by Liu *et al.* [1]. Galopin *et al.* [2] take electric scalar and magnetic vector potentials as variables. In the harmonic case, the ME effect is modified because of the resonance of the mechanical structure [3] and the occurrence of electromagnetic coupling based on Maxwell equations. In order to maintain an electric scalar potential formulation, the assumption of no magnetic induction in the direction normal to the working plane (z-direction) is performed by Nguyen *et al.* [4]. This assumption has to be justified because of the presence of electric field in piezoelectric elements creating a magnetic induction along z-direction.

The aim of this paper is to build a full 2D finite element formulation taking into account the electromagnetic coupling effect in order to discuss the assumptions of the previous model using magnetic and electric potentials [4]. A comparison is made on a pre-polarized bilayer. The corresponding constitutive law and finite element formulation are linearized around the operating point.

A bilayer and an asymmetric multilayer magnetic sensors are modeled by the 2D model using magnetic and electric potentials. Mechanical resonance frequencies are determined. The numerical results are compared with the experimental observations of Fetisov *et al.* [5].

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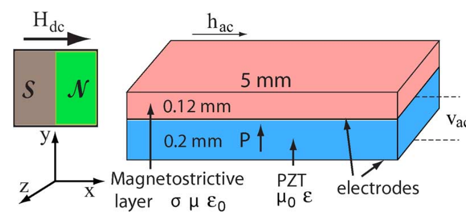

Fig. 1. Magnetoelectric bilayer.

II. Constitutive Laws

The working plane is defined in the xy-coordinates plane, z-direction is normal to the working plane (see Fig. 1). In the formulation, we denote the stress tensor by $\mathbf{T}$, the strain tensor by $\mathbf{S}$, the displacement by $\mathbf{u}$, the electric field by $\mathbf{E}$, the electric flux density by $\mathbf{D}$, the electric conductivity by σ, the magnetic field by $\mathbf{H}$, the magnetic induction by $\mathbf{B}$. We denote the small variation of X around a polarization point $\mathrm{X}_0(\mathrm{a}_0, \mathrm{b}_0)$ by $\widetilde{\mathrm{X}}(\widetilde{\mathrm{a}}, \widetilde{\mathrm{b}})$

$$\widetilde{X} = \frac{\partial X}{\partial a}(a_0, b_0)\widetilde{a} + \frac{\partial X}{\partial b}(a_0, b_0)\widetilde{b} \quad X = X_0 + \widetilde{X}. \qquad (1)$$

A. Linearization of Magnetostrictive and Piezoelectric Coefficients

1) Magnetostrictive Coefficients: We assume that the magnetostriction phenomenon is isochoric and isotropic. We consider that the magnetostriction strain $\mathbf{S}^{\mu}$ can be expressed as a parabolic function of the magnetization $\mathbf{M}$. We can then write [6]

$$s_{ij}^{\mu} = \frac{\beta_0}{2}\left(3m_i m_j - \delta_{ij}\|\mathbf{M}\|^2\right) \qquad (2)$$

where δ_{ij} is the Kronecker symbol, β_0 is the magnetostrictive coefficient, $\|\mathbf{M}\|$ is the norm of $\mathbf{M}$.

We also assume $\mathbf{M}$ and $\mathbf{B}$ to be collinear. The magnetostrictive strain can be written as follows [6]

$$s_{ij}^{\mu} = \frac{\beta_0}{2\mu_0^2}\left(3b_i b_j - \delta_{ij}\|\mathbf{B}\|^2\right)\frac{\|\mathbf{M}\|^2}{\|\mathbf{B}\|^2} \qquad (3)$$

where $\|\mathbf{B}\|$ is the norm of $\mathbf{B}$.

We consider that the polarization due to the applied static magnetic induction is along x-axis, thus the variation of magnetostrictive strain imposed by the variation of magnetic induction

$\mathbf{B}$ ($\mathbf{B} = B_0\mathbf{x} + \widetilde{\mathbf{B}}$) can be calculated as

$$\begin{aligned}\widetilde{\mathbf{S}}^{\mu}_{vec} &= \gamma(2\widetilde{b_x} \quad -\widetilde{b_x} \quad -\widetilde{b_x} \quad 0 \quad 3\widetilde{b_z} \quad 3\widetilde{b_y})^t\\ \gamma &= \frac{\beta_0}{2\mu_0^2}\left(\frac{\|\mathbf{M}\|^2}{\|\mathbf{B}\|^2}B_0 + \frac{\partial\frac{\|\mathbf{M}\|^2}{\|\mathbf{B}\|^2}}{\partial\|\mathbf{B}\|^2}B_0^2\right)\end{aligned} \quad (4)$$

where $\mathbf{S}_{vec}$ denotes the Voigt notation of the strain $\mathbf{S}$

$$\mathbf{S}_{vec} = (s_{xx} \quad s_{yy} \quad s_{zz} \quad 2s_{yz} \quad 2s_{zx} \quad 2s_{xy})^t. \quad (5)$$

2) Piezoelectric Coefficients: We consider that the polarization of the piezoelectric element is along y-axis. Using electrostrictive material, the electrostrictive deformation is a quadratic function of the electric induction. A similar expression of piezoelectric strain $\mathbf{S}^p$ imposed by the variation of electric displacement field $\mathbf{D}$ ($\mathbf{D} = D_0\mathbf{y} + \widetilde{\mathbf{D}}$) can be deduced

$$\widetilde{\mathbf{S}}^{p}_{vec} = D_0\alpha_0(-\widetilde{d_y} \quad 2\widetilde{d_y} \quad -\widetilde{d_y} \quad 3\widetilde{d_z} \quad 0 \quad 3\widetilde{d_x})^t. \quad (6)$$

B. Mechanical Assumptions

Using Lamé coefficients μ^* and λ^* in the case of isotropic material, the total stress $\mathbf{T}$ is expressed by

$$\mathbf{T} = 2\mu^*\mathbf{S}^e + \lambda^*\mathrm{tr}(\mathbf{S}^e)\mathbf{I} \quad (7)$$

where $\mathbf{S}^e = \mathbf{S} - \mathbf{S}^c$ is the elastic strain, $\mathbf{S}^c = \mathbf{S}^\mu$ in the magnetostrictive material (MM), $\mathbf{S}^c = \mathbf{S}^p$ in the piezoelectric material (PM). $\mathbf{I}$ is the identity second order tensor. In this paper we consider plane stress conditions ($t_{zx} = t_{zy} = t_{zz} = 0$) leading to the following relations

$$\begin{cases} s^e_{zx} = s^e_{zy} = 0\\ s^e_{zz} = \frac{\lambda^*}{2\mu^*+\lambda^*}\left(s^e_{xx} + s^e_{yy}\right)\end{cases}. \quad (8)$$

From (4), (6) and (8), the strain along z-direction can be calculated from the strain in the working plane:

- In MM:

$$\begin{aligned}\widetilde{s}_{zx} &= \frac{3}{2}\gamma\widetilde{b}_z\\ \widetilde{s}_{zy} &= 0\\ \widetilde{s}_{zz} &= \frac{\lambda^*}{2\mu^*+\lambda^*}(\widetilde{s}_{xx} + \widetilde{s}_{yy}) - \frac{2\mu^*}{2\mu^*+\lambda^*}B_0\beta_0\widetilde{b}_x\end{aligned} \quad (9)$$

- In PM:

$$\begin{aligned}\widetilde{s}_{zx} &= 0\\ \widetilde{s}_{zy} &= \frac{3}{2}D_0\alpha_0\widetilde{d}_z\\ \widetilde{s}_{zz} &= \frac{\lambda^*}{2\mu^*+\lambda^*}(\widetilde{s}_{xx} + \widetilde{s}_{yy}) - \frac{2\mu^*}{2\mu^*+\lambda^*}D_0\alpha_0\widetilde{d}_y.\end{aligned} \quad (10)$$

C. Constitutive Laws

Considering $\mathbf{S}$, $\mathbf{D}$ and $\mathbf{B}$ as state variables, at a polarization point of PM or MM, using the thermodynamical approach of Besbes *et al.* [7], and calculating the differentials of Hooke's law [4], the linearized forms of the constitutive laws of PM and MM are defined in 3D as:

- In MM:

$$\begin{aligned}\widetilde{t}_{ij} &= C_{ijkl}\widetilde{s}_{kl} - \beta_{ijk}\widetilde{b}_k\\ \widetilde{h}_i &= -\beta_{ijk}\widetilde{s}_{jk} + \nu_{ij}\widetilde{b}_j\end{aligned} \quad (11)$$

- In PM:

$$\begin{aligned}\widetilde{t}_{ij} &= C_{ijkl}\widetilde{s}_{kl} - \alpha_{ijk}\widetilde{d}_k\\ \widetilde{e}_i &= -\alpha_{ijk}\widetilde{s}_{jk} + \epsilon^{-1}_{ij}\widetilde{d}_j\end{aligned} \quad (12)$$

where C_{ijkl} is the stiffness tensor, $\boldsymbol{\nu}$ the effective reluctivity, $\boldsymbol{\beta}$ the magnetostrictive coupling tensor, $\boldsymbol{\alpha}$ the piezoelectric coupling tensor, $\boldsymbol{\epsilon}$ the effective permittivity.

From (4), (6), the coupling tensors $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ can be deduced from α_0, γ and D_0 depending on the polarization points

$$\begin{aligned}\alpha &= D_0\alpha_0\mu^*\begin{bmatrix}0 & 0 & 0 & 0 & 0 & 3\\ -2 & 4 & -2 & 0 & 0 & 0\\ 0 & 0 & 0 & 3 & 0 & 0\end{bmatrix}\\ \beta &= \gamma\mu^*\begin{bmatrix}4 & -2 & -2 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 3\\ 0 & 0 & 0 & 0 & 3 & 0\end{bmatrix}.\end{aligned} \quad (13)$$

From the mechanical assumptions ((9) and (10)), mechanical unknowns along z can be calculated afterwards from mechanical unknows in xy-plane. Consequently, from (11) and (12), $\mathbf{E}$ and $\mathbf{H}$ can be calculated separately in xy-plane (//) and along $z(\perp)$

$$\begin{aligned}&\widetilde{e}_\perp = \epsilon^{-1}_\perp\widetilde{d}_\perp && \widetilde{h}_\perp = \nu_\perp\widetilde{b}_\perp\\ &\widetilde{\mathbf{E}}_{//} = \boldsymbol{\epsilon}^{-1}_{//}\widetilde{\mathbf{D}}_{//} - \boldsymbol{\alpha}_{//}\widetilde{\mathbf{S}}_{//} && \widetilde{\mathbf{H}}_{//} = \boldsymbol{\nu}_{//}\widetilde{\mathbf{B}}_{//} - \boldsymbol{\beta}_{//}\widetilde{\mathbf{S}}_{//}\\ &\widetilde{\mathbf{T}}_{//} = \mathbf{C}_{//}\widetilde{\mathbf{S}}_{//} - \boldsymbol{\alpha}_{//}\widetilde{\mathbf{D}}_{//} - \boldsymbol{\beta}_{//}\widetilde{\mathbf{B}}_{//}\end{aligned} \quad (14)$$

where $\epsilon_\perp$ and $\nu_\perp$ are the equivalent permittivity and reluctivity along z-axis. In PM: $\boldsymbol{\beta} = \mathbf{0}$. In MM: $\boldsymbol{\alpha} = \mathbf{0}$.

The equivalent coefficients are as follows:

- In MM:

$$\begin{aligned}\boldsymbol{\beta}_{//} &= \gamma\mu^*\begin{bmatrix}\frac{-8\mu^*-6\lambda^*}{2\mu^*+\lambda^*} & \frac{4\mu^*}{2\mu^*+\lambda^*} & 0\\ 0 & 0 & -3\end{bmatrix}\\ \boldsymbol{\nu}_{//} &= \begin{bmatrix}\nu - \frac{4(\mu^*)^2}{2\mu^*+\lambda^*}\gamma^2 & 0\\ 0 & \nu\end{bmatrix}\\ \nu_\perp &= \nu - \frac{9}{2}\mu^*\gamma^2.\end{aligned} \quad (15)$$

- In PM:

$$\begin{aligned}\boldsymbol{\alpha}_{//} &= D_0\alpha_0\mu^*\begin{bmatrix}0 & 0 & -3\\ \frac{4\mu^*}{2\mu^*+\lambda^*} & \frac{-8\mu^*-6\lambda^*}{2\mu^*+\lambda^*} & 0\end{bmatrix}\\ \boldsymbol{\epsilon}^{-1}_{//} &= \begin{bmatrix}\epsilon^{-1} & 0\\ 0 & \epsilon^{-1} - \frac{4(\mu^*)^2}{2\mu^*+\lambda^*}D_0^2\alpha_0^2\end{bmatrix}\\ \epsilon^{-1}_\perp &= \epsilon^{-1} - \frac{9}{2}\mu^* D_0^2\alpha_0^2.\end{aligned} \quad (16)$$

III. Finite Element Formulation

A. Electromagnetic Equations

3D magnetoelectric coupling problems are governed by Maxwell equations

$$\mathbf{rot}\ \mathbf{E} = -\partial_t\mathbf{B} \quad \mathrm{rot}\ \mathbf{H} = \sigma\mathbf{E} + \partial_t\mathbf{D} \quad (17)$$

In the model proposed by Nguyen *et al.* [6], the magnetic induction along z-direction and the term $\partial_t\mathbf{D}$ have been neglected. Moreover, the magnetic induction and electric field in the working plane are assumed not to depend on z.

From $\mathbf{B} = \mathbf{rot}\ \mathbf{a}$, we can deduce that the magnetic vector potential is only along z.

As $b_z = 0$, equation $\mathbf{rot}\ \mathbf{E} = -\partial_t \mathbf{B}$ leads to consider $\mathbf{E} = \mathbf{grad}_{//} V$ where $\mathbf{grad}_{//}$ is defined in the working plane

$$\mathbf{grad}_{//} X = \begin{pmatrix} \dfrac{\partial X}{\partial x} & \dfrac{\partial X}{\partial y} \end{pmatrix}^t. \quad (18)$$

In this model, we suppose that $\mathbf{E}$ and $\mathbf{H}$ do not depend on z, thus each (17) can be divided into 2 equations

$$\begin{aligned} &\mathbf{r}^* \mathbf{grad}_{//} e_\perp = -j\omega \mathbf{B}_{//} \quad \mathbf{r}^* \mathbf{grad}_{//} h_\perp = \sigma \mathbf{E}_{//} + j\omega \mathbf{D}_{//} \\ &\mathrm{div}_{//}(\mathbf{r}^* \mathbf{E}_{//}) = -j\omega b_\perp \quad \mathrm{div}_{//}(\mathbf{r}^* \mathbf{H}_{//}) = \sigma e_\perp + j\omega d_\perp \end{aligned} \quad (19)$$

where $\mathbf{r}^*$ is defined as $\mathbf{r}^* = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and the operator $\mathrm{div}_{//}$ is defined in the working plane, meaning that

$$\mathrm{div}_{//} \mathbf{X} = \frac{\partial X_x}{\partial x} + \frac{\partial X_y}{\partial y}. \quad (20)$$

Noting $a^* = -(e_\perp / j\omega)$ and $t^* = h_\perp / j\omega$, $\mathbf{I}$ the identity matrix, using the constitutive laws (14), the system to be solved is

$$\begin{aligned} &\mathrm{div}_{//}\left(\mathbf{r}^* \left(\boldsymbol{\epsilon}_{//} + \frac{\sigma}{j\omega} \mathbf{I} \right)^{-1} \mathbf{r}^* \mathbf{grad}_{//} t^* - \mathbf{r}^* \boldsymbol{\alpha}_{//}^t \mathbf{S}_{//} \right) = \omega^2 \boldsymbol{\nu}_\perp^{-1} t^* \\ &\mathrm{div}_{//}\left(\mathbf{r}^* \boldsymbol{\nu}_{//} \mathbf{r}^* \mathbf{grad}_{//} a^* - \mathbf{r}^* \boldsymbol{\beta}_{//}^t \mathbf{S}_{//} \right) \\ &\quad = (-j\omega\sigma + \omega^2 \epsilon_\perp) a^*. \end{aligned} \quad (21)$$

B. Mechanical Equation

The mechanical equilibrium reads

$$\mathrm{div}\mathbf{T} + \mathbf{F} = \rho_m \frac{\partial^2 \mathbf{u}}{\partial t^2} \quad (22)$$

where $\mathbf{F}$ is the volume force, ρ_m the mass density and $\mathbf{u}$ the displacement. From (14) and (19) the mechanical equation to be solved in 2D is

$$\begin{aligned} &\mathrm{div}(\mathbf{C}_{//} \mathbf{S}_{//} - \boldsymbol{\alpha}_{//} \mathbf{r}^* \mathbf{grad} t^*) = -\mathbf{F} + \rho_m \frac{\partial^2 \mathbf{u}}{\partial t^2} (\mathrm{PM}) \\ &\mathrm{div}(\mathbf{C}_{//} \mathbf{S}_{//} - \boldsymbol{\beta}_{//} \mathbf{r}^* \mathbf{grad} a^*) = -\mathbf{F} + \rho_m \frac{\partial^2 \mathbf{u}}{\partial t^2} (\mathrm{MM}) \end{aligned} \quad (23)$$

where $\mathbf{C}_{//}$ is the equivalent elastic matrix

$$\mathbf{C}_{//} = \begin{bmatrix} \frac{4(\mu^*)^2 + 4\mu^*\lambda^*}{2\mu^* + \lambda^*} & \frac{2\mu^*\lambda^*}{2\mu^* + \lambda^*} & 0 \\ \frac{2\mu^*\lambda^*}{2\mu^* + \lambda^*} & \frac{4(\mu^*)^2 + 4\mu^*\lambda^*}{2\mu^* + \lambda^*} & 0 \\ 0 & 0 & \mu^* \end{bmatrix}. \quad (24)$$

As $\mathbf{S}_{//} = (1/2)(\mathbf{grad}\ \mathbf{u}_{//} + {}^t\mathbf{grad}\ \mathbf{u}_{//})$, the variables of the system to be solved are $\mathbf{u}_{//}, t^*, a^*$. A damping term is added in the mechanical finite element formulation, the damping coefficient is 2%. Compared to the $(\mathbf{u}_{//}, V, \mathbf{a})$ formulation using the mechanical displacement $\mathbf{u}$, the electric and magnetic potentials V and $\mathbf{a}$ proposed in [4], the full $(\mathbf{u}_{//}, t^*, a^*)$ formulation developped in this paper has the same computation cost: 4 unknowns per node.

IV. Numerical Example

In this section, we study a ME bilayer (Fig. 1) pre-polarized by a static magnetic field $\mathbf{H}_{\mathrm{dc}}$. By adding a harmonic magnetic field $\mathbf{h}_{\mathrm{ac}}$ at mechanical resonance frequency, the electric voltage v_{ac} at piezoelectric electrodes can be amplified and returns information concerning the static magnetic field [8]–[10]. The left side of the bilayer is clamped. The dynamic magnetic field is generated with Dirichlet conditions on a^* imposed on upper and lower boundaries.

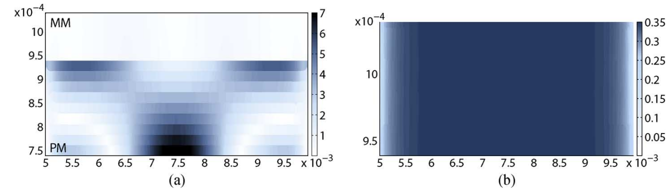

Fig. 2. Formulation using $(\mathbf{u}_{//}\mathbf{V}, \mathbf{a})$ as variables, $\mathbf{H}_{\mathrm{dc}} = 5$ Oe. (a) Modulus of electric field in the xy-plane $(PM + MM)$; (b) modulus of magnetic induction in the xy-plane (*MM only*).

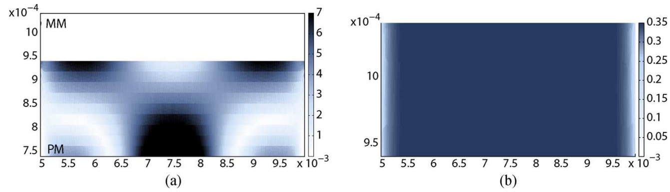

Fig. 3. Formulation using $(\mathbf{u}_{//}, t^*, a^*)$ as variables, $\mathbf{H}_{\mathrm{dc}} = 5$ Oe. (a) Modulus of electric field in the xy-plane $(PM + MM)$; (b) modulus of magnetic induction in the xy-plane (*MM only*).

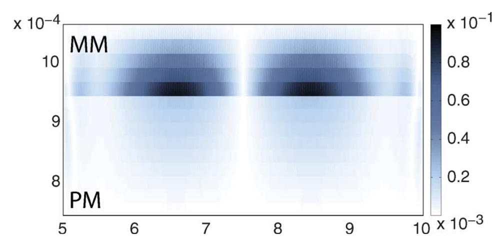

Fig. 4. Magnetic induction along z-direction $B_\perp$ (PM + MM).

The MM is an electrical conductor with electrical conductivity $\sigma = 2 * 10^{-6}$ S/m. The applied static magnetic field is $\mathbf{H}_{\mathrm{dc}} = 5$ Oe. The working frequency is 100 kHz corresponding to the maximum working frequency for the sensor. By imposing the equipotential condition on electrodes, the $(\mathbf{u}_{//}, V, \mathbf{a})$ model allows to get easily the electric voltage on electrodes. In order to consider electrodes with the $(\mathbf{u}_{//}, t^*, a^*)$ model, nodes on electrodes are duplicated. A post-processing is then necessary to get the value of electric voltage. In this paper, the electric voltage for both approaches has not been compared.

Fig. 2 shows the electric field and magnetic induction obtained using the $(\mathbf{u}, V, \mathbf{a})$ model.

The result obtained with the full $(\mathbf{u}_{//}, t^*, a^*)$ 2D-model under similar conditions is presented in Fig. 3.

The two results are slightly different. A quite uniform magnetic induction in MM is obtained with both 2D models. The electric field in the working plane is lower in the first model than in the full model. The existence of the electric field in the working plane means that there is a component of magnetic induction along z. It can be calculated using the full model. The result is presented in Fig. 4.

The result in Fig. 4 confirms the existence of a magnetic induction along z-axis in PM. Nevertheless, its value is negligible

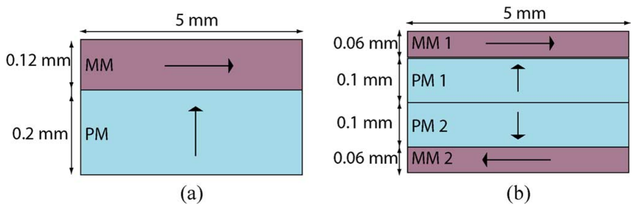

Fig. 5. Composite structures for magnetic sensors. (a) Initial sensor; (b) asymmetric sensor.

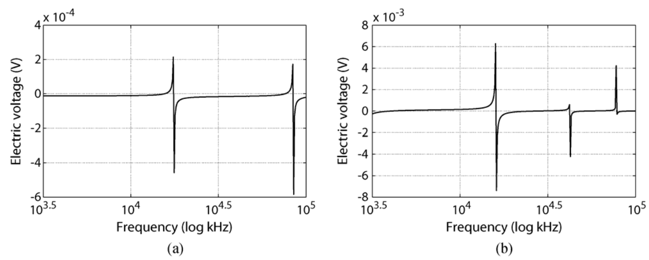

Fig. 6. Evolution of electric voltage as function of frequency, $\mathbf{H}_{\mathrm{dc}} = 5\ \mathrm{Oe}$. (a) Initial sensor; (b) asymmetric sensor.

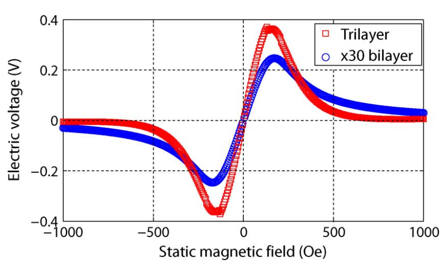

Fig. 7. Evolution of electric voltage as function of magnetic static field ($f = 17\ \mathrm{kHz}$).

compared to the magnetic induction in xy-plane (less than 0.1% of in-plane induction). Considering no magnetic induction along z-axis is therefore a justified assumption in this range of frequency. The first model will then be used in the following.

As an application, bilayer and multilayer sensors are studied. The electrodes are equipotential surfaces imposed on the piezoelectric material. The corresponding configurations are shown in Fig. 5.

The structure presented in Fig. 5(b) is asymmetric. MM1 and MM2 are materials with respectively negative and positive magnetostriction. Both structures facilitate bending oscillations. In asymmetric sensor, the piezoelectric part is divided into two layers with opposite polarization. This arrangement of piezoelectric materials leads to higher electric voltage [5].

First we determine the resonance frequency for both structures. The result is given in Fig. 6.

The first resonance frequency obtained is close to 17 kHz for both structures. For this frequency, Fig. 7 plots the electric voltage as a function of the static magnetic field $\mathbf{H}_{\mathrm{dc}}$. The change of static magnetic field $\mathbf{H}_{\mathrm{dc}}$ modifies the coupling coefficient γ and the equivalent reluctivity $\boldsymbol{\nu}$ in the magnetostrictive material, leading to variable electric voltage.

In Fig. 7, the electric voltage obtained for the bilayer is amplified 30 times to have the same order of magnitude as the electric voltage obtained for the multilayer. The electric voltage obtained with the asymmetric multilayer is much higher than for the bilayer. The result is in good qualitative agreement with the experimental results obtained by Fetisov *et al.* [5], Giang *et al.* [9], Vopsaroiu *et al.* [11] and Quandt *et al.* [12].

V. Conclusion

A 2D finite element model using $(\mathbf{u}_{//}, t^*, a^*)$ as variables is built for the modeling of extrinsic ME effect. The model is applied to the prediction of the ME coefficient of magnetic sensors. It is shown that the component of the magnetic and electric field normal to the plane of the sensors can be neglected. The advantage of using a 2D model with such assumption is the fact that the electric voltage on electrodes can be obtained easily. An application is proposed for the modeling of bilayer and multilayer configurations for magnetic field sensors. The resonance frequency of both structures are very close but a higher electric voltage is obtained using the multilayer. The model allows to capture the effect and reproduce a similar shape than experimental results excerpted from the literature. A next step will be a quantitative validation. Material parameters have to be accurately identified for that purpose.

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