

Finite element modeling of magnetic field sensors based on nonlinear magnetolectric effect

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Magnetolectric effect in composite materials results from the combination of piezoelectric and magnetostrictive effects. This paper focuses on the development of a harmonic finite element formulation for such coupled problems, taking into account the nonlinearity of magnetostrictive behavior. An application to a magnetic sensor operating under dynamic excitation is presented in order to illustrate the formulation. The enhancement of the magnetolectric coefficient when a low amplitude harmonic field is superimposed to the static field to be measured is shown to be related to the nonlinearity of magnetic and magnetostrictive behavior. © 2011 American Institute of Physics. [doi:10.1063/1.3553855]

I. INTRODUCTION

Magnetolectricity consists in the coupling between magnetic and electric fields even under static conditions.¹ The magnetolectric (ME) effect can be either intrinsic, in single phase materials, or extrinsic, in composite materials. Up to now, the highest magnetolectric coefficients have been observed for extrinsic ME effect. We consider here only the extrinsic effect resulting from the combination of magnetostrictive and piezoelectric effects. In composite materials, this ME effect can be written in a simple way either in the form of Eqs. (1) and (2):²

$$ME_1 = \frac{\text{electrical}}{\text{mechanical}} \cdot \frac{\text{mechanical}}{\text{magnetic}}, \quad (1)$$

$$ME_2 = \frac{\text{magnetic}}{\text{mechanical}} \cdot \frac{\text{mechanical}}{\text{electrical}}. \quad (2)$$

Recently, research activities on ME composite materials have increased rapidly,^{1,3} but numerical approaches are still limited due to the lack of a complete theoretical formulation. A unified formulation is required to deal with nonlinearity and frequency dependence of the ME effect. Up to now, several numerical models have been developed: The frequency effect is integrated in the linear model of Liu *et al.*,⁴ but only from the mechanical point of view, and the model of Galopin *et al.*⁵ takes into account the nonlinear behavior of the magnetostrictive phase, but only under quasistatic loadings. Recently, a nonlinear dynamic scalar formulation has been proposed.⁶

The objective of this paper is to build a numerical vector model based on the finite element method, accounting for nonlinearities of both magnetostrictive and magnetic behavior. Magnetostriction is described as a quadratic function of magnetization. Constitutive laws coupling mechanical/electric/magnetic effects are first detailed, and then introduced into a finite element formulation. An application to the mod-

eling of a ME sensor under different configurations is finally proposed. For such an application, it has been observed that the sensor sensitivity is enhanced when a harmonic field at the resonance frequency is superimposed to the static field to be measured.^{7,8} The work reported in this paper proves that this enhanced sensitivity is directly related to the nonlinearity of magnetic and magnetostrictive behavior.

II. EQUILIBRIUM EQUATIONS

The variables used in the model are the stress tensor **T**, the strain tensor **S**, the displacement **u**, the volumic force **f**, the mass density ρ_m , the electric field **E**, the electric flux density **D**, the electrical voltage *V*, the charge density ρ , the electric conductivity σ , the magnetic field **H**, the magnetic induction **B**, the magnetic vector potential **a**, and the current density **J**. For a vector **X**, we note x_i the components of **X**.

A. Mechanical equilibrium

The mechanical equilibrium is given by

$$\text{div } \mathbf{T} + \mathbf{f} = \rho_m \frac{\partial^2 \mathbf{u}}{\partial t^2}. \quad (3)$$

Noting \mathbf{S}^e the elastic strain, we consider a 2D problem with plane stress conditions ($t_{31} = t_{32} = t_{33} = 0$) leading to the following relations:

$$\begin{aligned} s_{31}^e &= s_{32}^e = 0 \\ s_{33}^e &= \frac{\lambda^*}{2\mu^* + \lambda^*} (s_{11}^e + s_{22}^e). \end{aligned} \quad (4)$$

As s_{33}^e can be calculated afterwards, we only have to consider the mechanical strains in the working plane. In the finite element form, the mechanical variable chosen is then the displacement **u** in the working plane:

$$\mathbf{S} = \frac{1}{2} (\mathbf{grad } \mathbf{u} + \mathbf{grad}^t \mathbf{u}) = \mathbf{D}\mathbf{u}. \quad (5)$$

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B. Electromagnetic equilibrium

The electromagnetic equations are given by the standard magnetodynamic Maxwell's equations (neglecting the displacement currents):

$$\mathbf{curl}(\mathbf{H}) = \mathbf{J}, \quad (6)$$

$$\mathbf{div} \mathbf{D} = \rho, \quad (7)$$

$$\mathbf{div} \mathbf{B} = 0. \quad (8)$$

Moreover, at the frequencies and geometry considered in this paper (see Sec. V), eddy currents can be neglected because the skin depth calculated from analytical equation is much higher than the usual depth of magnetostrictive layers.⁶ From Eq. (8), magnetic induction \mathbf{B} can be written as: $\mathbf{B} = \mathbf{curl}(\mathbf{a})$, where \mathbf{a} is the magnetic vector potential.

We consider a 2D problem with the following assumptions: Neither magnetic induction \mathbf{B} , nor electric field \mathbf{E} , are considered in the direction perpendicular to the working plane (z direction). Moreover, \mathbf{B} and \mathbf{E} are assumed to be invariant with z . These assumptions lead to the following simplifications.

- (i) The magnetic vector potential is along the z direction and independent of z ($a_1 = a_2 = 0$).
- (ii) The electric field in the working plane $E_{//}$ can be written: $\mathbf{E}_{//} = \mathbf{grad} V(x, y)$.

The electric voltage V and the magnetic vector potential along the z direction a_3 are chosen as the electromagnetic variables in the finite element formulation.

III. FINITE ELEMENT FORMULATION

Finite element method (FEM) is one of the most widespread tools used to solve the partial differential equations such as the given equilibrium equations (3), (6), and (7). The FEM is usually associated with the variational methods or residual methods. The variational method for such a coupled problem has been presented by Galopin *et al.*⁵ by minimizing the energy function. The residual methods directly solve the equilibrium equations. It is an advantage compared to the variational methods, especially under harmonic loadings when the energy functions are not easy to determine. In this paper, a particular residual method—the Galerkin method—is chosen to establish the 2D finite element formulation. This formulation is well suited to our electro-magneto-elastic coupled problem. The constitutive law in the coupled problem can be written in the generic matrix form of the following equation [the caret denotes the use of Voigt notation (see Appendix A)]:

$$\begin{pmatrix} \hat{\mathbf{T}} \\ \mathbf{D} \\ \mathbf{H} \end{pmatrix} = \begin{bmatrix} \hat{c} & -\hat{e}^t & -\hat{q}^t \\ \hat{e} & \epsilon & \alpha^t \\ -\hat{q} & \alpha & \nu \end{bmatrix} \begin{pmatrix} \hat{\mathbf{S}} \\ \mathbf{E} \\ \mathbf{B} \end{pmatrix}. \quad (9)$$

In the case of piezoelectric material, the constitutive law is expressed by

$$\begin{pmatrix} \hat{\mathbf{T}} \\ \mathbf{D} \\ \mathbf{H} \end{pmatrix} = \begin{bmatrix} \hat{c} & -\hat{e}^t & 0 \\ \hat{e} & \epsilon & 0 \\ 0 & 0 & \nu \end{bmatrix} \begin{pmatrix} \hat{\mathbf{S}} \\ \mathbf{E} \\ \mathbf{B} \end{pmatrix}. \quad (10)$$

In the case of magnetostrictive material, the constitutive law is expressed by:

$$\begin{pmatrix} \hat{\mathbf{T}} \\ \mathbf{D} \\ \mathbf{H} \end{pmatrix} = \begin{bmatrix} \hat{c} & 0 & -\hat{q}^t \\ 0 & \epsilon & 0 \\ -\hat{q} & 0 & \nu \end{bmatrix} \begin{pmatrix} \hat{\mathbf{S}} \\ \mathbf{E} \\ \mathbf{B} \end{pmatrix}. \quad (11)$$

The coefficients ϵ and \mathbf{q} are, respectively, the piezoelectric and magnetostrictive coefficients.

A. Mechanical formulation

The mechanical formulation is established by taking into account the mechanical equilibrium (3) and the constitutive law deduced from Eq. (9):

$$\mathbf{T} = \mathbf{c} : \mathbf{S} - \mathbf{e}^t \cdot \mathbf{E} - \mathbf{q}^t \cdot \mathbf{B}.$$

Noting Ω the study domain, Γ_s its boundaries, the weak formulation of the mechanical equilibrium equation is

$$\int_{\Omega} \mathbf{w} \cdot \left(\mathbf{div} \mathbf{T} + \mathbf{f} - \rho_m \frac{\partial^2 \mathbf{u}}{\partial t^2} \right) d\Omega = 0, \quad (12)$$

where \mathbf{w} is a vectorial test function. Integrating by parts, Eq. (12) leads to

$$\int_{\Omega} \left(D\mathbf{w} : \mathbf{T} - \mathbf{w} \cdot \mathbf{f} + \rho_m \mathbf{w} \cdot \frac{\partial^2 \mathbf{u}}{\partial t^2} \right) d\Omega = \oint_{\Gamma_s} \mathbf{T} \cdot \mathbf{n} \cdot \mathbf{w} d\Gamma \quad (13)$$

with \mathbf{n} the normal vector to the boundary Γ_s . The term on the right-hand side of Eq. (13) is related to the boundary conditions. There are two types of boundary conditions we need to contend with.

- (1) Dirichlet boundary condition: The values of \mathbf{u} are known on a part of the boundary Γ_s .
- (2) Neumann condition: $\mathbf{T} \cdot \mathbf{n} = 0$, the right-hand side of Eq. (13) is zero.

Replacing $\mathbf{S} = D\mathbf{u}$, $\mathbf{E} = \mathbf{grad} V$ and $\mathbf{B} = \mathbf{curl}(\mathbf{a})$, Eq. (13) can be written as

$$\begin{aligned} & \int_{\Omega} D\mathbf{w} : \mathbf{c} : D\mathbf{u} d\Omega - \int_{\omega} D\mathbf{w} : \mathbf{e}^t \cdot \mathbf{grad} V d\Omega \\ & - \int_{\Omega} D\mathbf{w} : \mathbf{q}^t \cdot \mathbf{curl}(\mathbf{a}) d\Omega + \int_{\Omega} \rho_m \mathbf{w} \cdot \frac{\partial^2 \mathbf{u}}{\partial t^2} d\Omega \\ & = \int_{\Omega} \mathbf{w} \cdot \mathbf{f} d\Omega + \oint_{\Gamma_s} \mathbf{T} \cdot \mathbf{n} \cdot \mathbf{w} d\Gamma. \end{aligned} \quad (14)$$

In our mechanical problem no body force is considered ($\mathbf{f} = \mathbf{0}$). The magnetic vector potential \mathbf{a} is along z direction, therefore: $\mathbf{curl}(\mathbf{a}) = \mathbf{r}^* \cdot \mathbf{grad} a_3$ with

$$\mathbf{r}^* = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$$

In the finite element formulation, the displacement \mathbf{u} , the electric potential V , and the vector potential a_3 over an element are related to the corresponding node values $\{\mathbf{u}\}$, $\{V\}$, and $\{a_3\}$ using the shape functions $[\mathbf{w}]$, $[N_v]$, and $[N_a]$:

$$\begin{aligned}\mathbf{u} &= [\mathbf{w}]\{\mathbf{u}\} \\ V &= [N_V]\{V\} \\ a_3 &= [N_a]\{a_3\}.\end{aligned}\quad (15)$$

Therefore, the strain \mathbf{S} , the electrical field \mathbf{E} , and the magnetic induction \mathbf{B} are associated to the nodal displacements and potentials by the derivatives $[G_u]$, $[G_v]$, and $[G_a]$ of the shape functions:

$$\begin{aligned}\hat{\mathbf{S}} &= D[\mathbf{w}]\{\mathbf{u}\} = [G_u]\{\mathbf{u}\} \\ \mathbf{E} &= \mathbf{grad}[N_V]\{V\} = [G_V]\{V\} \\ \mathbf{B} &= \mathbf{r}^* \mathbf{grad}[N_a]\{a_3\} = [\mathbf{r}^*][G_a]\{a_3\}.\end{aligned}\quad (16)$$

After discretization, Eq. (14) can be written in the matrix form:

$$([\mathbb{K}_{uu}] - \omega^2[\mathbb{M}])\{\mathbf{u}\} + [\mathbb{K}_{up}]\{V\} + [\mathbb{K}_{ua}]\{a_3\} = [\mathbf{0}] \quad (17)$$

with

$$\begin{aligned}[\mathbb{K}_{uu}] &= \sum_e \int_{\Omega^e} [G_u]^t [\hat{\mathbb{C}}] [G_u] d\Omega \\ [\mathbb{M}] &= \sum_e \int_{\Omega^e} \rho_m [\mathbf{w}]^t [\mathbf{w}] d\Omega \\ [\mathbb{K}_{up}] &= - \sum_e \int_{\Omega^e} [G_u]^t [\hat{\mathbb{C}}^t] [G_V] d\Omega \\ [\mathbb{K}_{ua}] &= - \sum_e \int_{\Omega^e} [G_u]^t [\hat{\mathbb{C}}^t] [\mathbf{r}^*] [G_a] d\Omega\end{aligned}\quad (18)$$

where Ω^e is the partial domain associated to the mesh element e . Equation (17) can be complemented with a damping term $j\omega\alpha[\mathbb{K}_{uu}]\{\mathbf{u}\}$ with α the damping coefficient. Noting $[\mathbb{K}_{uu}^*] = [\mathbb{K}_{uu}] + j\omega\alpha[\mathbb{K}_{uu}] - \omega^2[\mathbb{M}]$ gives:

$$[\mathbb{K}_{uu}^*]\{\mathbf{u}\} + [\mathbb{K}_{up}]\{V\} + [\mathbb{K}_{ua}]\{a_3\} = [\mathbf{0}] \quad (19)$$

B. Electromagnetic formulation

In a similar way, equations $\text{div } \mathbf{D} = \rho$ and $\text{curl}(\mathbf{H}) = \mathbf{J}$ gives the following expressions:

$$\begin{aligned}[\mathbb{K}_{up}]^t \{\mathbf{u}\} + [\mathbb{K}_{pp}]\{V\} &= [\mathcal{Q}] + [\mathcal{Q}_n] \\ [\mathbb{K}_{ua}]^t \{\mathbf{u}\} + [\mathbb{K}_{aa}]\{a_3\} &= [\mathcal{I}] + [\mathcal{I}_n]\end{aligned}\quad (20)$$

with

$$\begin{aligned}[\mathbb{K}_{pp}] &= \sum_e \int_{\Omega^e} [G_V]^t [\hat{\mathbb{E}}] [G_V] d\Omega \\ [\mathcal{Q}] &= \sum_e \int_{\Omega^e} \rho [N_V]^t d\Omega \\ [\mathcal{Q}_n] &= \sum_e \oint_{\Gamma_s} D_n [N_V]^t d\Gamma \\ [\mathbb{K}_{aa}] &= \sum_e \int_{\Omega^e} [G_a]^t [\mathbf{r}^*]^t [\hat{\mathbb{V}}] [\mathbf{r}^*] [G_a] d\Omega \\ [\mathcal{I}] &= \sum_e \int_{\Omega^e} j_3 [N_V]^t d\Omega \\ [\mathcal{I}_n] &= \sum_e \oint_{\Gamma_s} H_t [N_V]^t d\Gamma\end{aligned}\quad (21)$$

where $\hat{\mathbb{V}}$ is the equivalent reluctivity tensor accounting for the magnetoelastic coupling (see Sec. IV B 2), D_n is the component of \mathbf{D} normal to Γ_s , j_3 is the electric current along the z direction, and H_t is the component of \mathbf{H} tangent to Γ_s . In the case of the sensor studied in the following we will consider no electric charges ($\rho = 0$) and no current density (so that $j_3 = 0$). We finally obtain the following system:

$$\begin{bmatrix} \mathbb{K}_{uu}^* & \mathbb{K}_{up} & \mathbb{K}_{ua} \\ \mathbb{K}_{pu} & \mathbb{K}_{pp} & \mathbf{0} \\ \mathbb{K}_{au} & \mathbf{0} & \mathbb{K}_{aa} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ V \\ a_3 \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{0} \\ I_n \end{Bmatrix}, \quad (22)$$

where $\mathbb{K}_{pu} = \mathbb{K}_{up}^t$ describes the electro-mechanical coupling, $\mathbb{K}_{au} = \mathbb{K}_{ua}^t$ describes the magnetomechanical coupling. Linear system (22) is solved using Gauss algorithm.

IV. CONSTITUTIVE LAWS

ME composite materials often consist in an assembly of piezoelectric (pz) and magnetostrictive (ms) materials. The electroelastic behavior is assumed to be linear. The magnetostrictive behavior is nonlinear.

A. Electroelastic behavior

Considering that the piezoelectric material is prepolarized, the constitutive law is assumed to be linear around the polarization point:

$$\begin{pmatrix} \tilde{\mathbf{T}} \\ \tilde{\mathbf{D}} \end{pmatrix} = \begin{bmatrix} \tilde{\mathbb{C}} & -\tilde{\mathbb{C}}^t \\ \tilde{\mathbb{E}} & \tilde{\mathbb{E}} \end{bmatrix}_{pz} \begin{pmatrix} \tilde{\mathbf{S}} \\ \tilde{\mathbf{E}} \end{pmatrix}. \quad (23)$$

$\tilde{X}(\tilde{a}, \tilde{b})$ —applied to a mechanical, electric or magnetic field—denotes the small variation of X around a polarization point $X_0(a_0, b_0)$:

$$\tilde{X} = \frac{\partial X}{\partial a}(a_0, b_0)\tilde{a} + \frac{\partial X}{\partial b}(a_0, b_0)\tilde{b}, \quad X = X_0 + \tilde{X}. \quad (24)$$

B. Magnetostrictive behavior

1. General form

The magnetostrictive material is not prepolarized, therefore its constitutive law is strongly nonlinear and has then to be investigated. The total strain \mathbf{S} is divided⁹ into the elastic strain \mathbf{S}^e and the magnetostriction strain \mathbf{S}^μ , $\mathbf{S} = \mathbf{S}^e + \mathbf{S}^\mu$. According to Hooke's law, the total stress is expressed by

$$t_{ij} = C_{ijkl}^{ms} (s_{kl} - s_{kl}^\mu) = C_{ijkl}^{ms} s_{kl} - t_{ij}^\mu, \quad (25)$$

where C_{ijkl}^{ms} is the usual stiffness tensor of the magnetostrictive material under static loading.

In the case of an isotropic material, Eq. (25) can be written using Lamé coefficients μ^* and λ^* :

$$t_{ij} = 2\mu^* (s_{ij} - s_{ij}^\mu) + \delta_{ij}\lambda^* (s_{kk} - s_{kk}^\mu), \quad (26)$$

where δ_{ij} is the Kronecker symbol ($\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$ if $i \neq j$).

The magnetostriction phenomenon is assumed to be isochoric¹⁰ (meaning $s_{kk}^\mu = 0$) and isotropic. Magnetostriction strain is also assumed to be a parabolic function of the magnetization. Modifying the model of Galopin *et al.*,⁵ who considered the magnetostriction strain \mathbf{S} as a parabolic function of the magnetic induction \mathbf{B} , and assuming that \mathbf{B} and \mathbf{M} are collinear, the magnetostriction strain can be expressed by

$$s_{ij}^\mu = \frac{\beta}{2\mu_0^2} (3b_i b_j - \delta_{ij} b_k b_k) \frac{\mathbf{m}^2}{\mathbf{b}^2}, \quad (27)$$

where $\mathbf{m}^2 = m_i m_i$, $\mathbf{b}^2 = b_i b_i$, and β is deduced from experimental results.¹¹

Using the thermodynamical approach of Besbes *et al.*¹² (writing $\partial t_{kl} / \partial b_i = \partial h_i / \partial s_{kl}$) with respect to the independent variables \mathbf{S} and \mathbf{B} , the magnetic field can be expressed as

$$h_i = v_{ij} b_j - \frac{\partial t_{ij}^\mu}{\partial b_i} (s_{kl} - s_{kl}^\mu), \quad (28)$$

where v_{ij} is the reluctivity tensor of the magnetostrictive material.

2. Linearized form

In order to describe the constitutive law at a polarization point of the magnetostrictive material, the differentials of Eqs. (25) and (28) are calculated, leading, respectively, to Eqs. (29) and (30):

$$\tilde{t}_{ij} = C_{ijkl}^{ms} \tilde{s}_{kl} - \frac{\partial t_{ij}^\mu}{\partial b_k} \tilde{b}_k, \quad (29)$$

$$\tilde{h}_i = -\frac{\partial t_{kl}^\mu}{\partial b_i} \tilde{s}_{kl} + \left[\frac{\partial v_{ik} b_k}{\partial b_j} - \frac{\partial^2 t_{kl}^\mu}{\partial b_i \partial b_j} (s_{kl} - s_{kl}^\mu) + \frac{\partial t_{kl}^\mu}{\partial b_i} \frac{\partial s_{kl}^\mu}{\partial b_j} \right] \tilde{b}_j. \quad (30)$$

Equation (30) introduces the terms to be calculated: $\partial t_{kl}^\mu / \partial b_i$ and $\partial v_{ik} b_k / \partial b_j$ correspond respectively to the coupling matrix \mathfrak{q} and the equivalent reluctivity \tilde{v}_{ij} . Two additional terms $(\partial^2 t_{kl}^\mu / \partial b_i \partial b_j) (s_{kl} - s_{kl}^\mu)$ and $(\partial t_{kl}^\mu / \partial b_i) (\partial s_{kl}^\mu / \partial b_j)$ also need to be estimated.

- (1) As $s_{kk}^\mu = 0$ (isochoric magnetostriction), the coupling matrix $\partial t_{kl}^\mu / \partial b_i$ can be calculated as follows in the case of isotropic elasticity:

$$\frac{\partial t_{kl}^\mu}{\partial b_i} = 2\mu^* \frac{\partial s_{kl}^\mu}{\partial b_i}. \quad (31)$$

Using Eq. (27) we obtain

$$\frac{\partial t_{kl}^\mu}{\partial b_i} = \frac{\mu^* \beta}{\mu_0^2} \left[\frac{\partial (3b_k b_l - \delta_{kl} b_j b_j)}{\partial b_i} \frac{\mathbf{m}^2}{\mathbf{b}^2} + (3b_k b_l - \delta_{kl} b_j b_j) \frac{\partial (\mathbf{m}^2 / \mathbf{b}^2)}{\partial \mathbf{b}} \frac{\partial \mathbf{b}}{\partial b_i} \right] \quad (32)$$

with

$$\frac{\partial \mathbf{b}}{\partial b_i} = \frac{b_i}{\mathbf{b}}$$

and

$$\frac{\partial (3b_k b_l - \delta_{kl} b_j b_j)}{\partial b_i} = \begin{cases} 4b_i & \text{if } k = l = i \\ -2b_i & \text{if } k = l \neq i \\ 3b_l & \text{if } k = i \neq l \\ 3b_k & \text{if } l = i \neq k \\ 0 & \text{else} \end{cases}$$

- (2) The nonlinear relationship between \mathbf{M} and \mathbf{H} is written using a Langevin-type equation:¹³

$$M = M_s \left[\coth(A_s H M_s) - \frac{1}{A_s H M_s} \right], \quad (33)$$

with M_s the saturation magnetization, the constant A_s can be defined as $A_s = 3\mu_0 \chi_0 / M_s^2$ with χ_0 the initial susceptibility of the anhysteretic magnetization curve.¹⁴ Replacing in Eq. (33) and using $B = \mu_0(H + M)$, it comes:

$$B = \mu_0 \left[H + M_s \left(\frac{1}{\tanh(3\mu_0 \chi_0 H / M_s)} - \frac{M_s}{3\mu_0 \chi_0 H} \right) \right]. \quad (34)$$

- (3) The first term of the equivalent reluctivity \tilde{v}_{ij} can be expressed as follows:

$$\frac{\partial v_{ik} b_k}{\partial b_j} = v_{ij} + \frac{\partial v_{ii}}{\partial \mathbf{b}} \frac{b_i b_j}{\mathbf{b}}. \quad (35)$$

Unlike the initial reluctivity that is diagonal ($v_{ij} \neq 0$ only if $i = j$), this expression introduces extra diagonal terms, which may not be negligible.

- (4) Additional terms $(\partial^2 t_{kl}^\mu / \partial b_i \partial b_j) (s_{kl} - s_{kl}^\mu)$ and $(\partial t_{kl}^\mu / \partial b_i) (\partial s_{kl}^\mu / \partial b_j)$ have to be estimated: Knowing $[\partial^2 (3b_k b_l - \delta_{kl} b_p b_p) / \partial b_i \partial b_j] (s_{kl} - s_{kl}^\mu) = 4(3s_{ij} - \delta_{ij} s_{kk})$, the term $(\partial^2 t_{kl}^\mu / \partial b_i \partial b_j) (s_{kl} - s_{kl}^\mu)$ is calculated from the derivation of Eq. (32). In the case of no applied strain, $s_{kl} - s_{kl}^\mu = 0$, therefore the term $(\partial^2 t_{kl}^\mu / \partial b_i \partial b_j) (s_{kl} - s_{kl}^\mu)$ vanishes. In a similar way, the term $(\partial t_{kl}^\mu / \partial b_i) (\partial s_{kl}^\mu / \partial b_j)$ can be calculated using Eq. (36):

$$\frac{\partial (3b_k b_l - \delta_{kl} b_p b_p)}{\partial b_i} \frac{\partial (3b_k b_l - \delta_{kl} b_p b_p)}{\partial b_j} = \begin{cases} 6(b_i^2 + 3\mathbf{b}^2) & \text{if } i=j \\ 6b_i b_j & \text{if } i \neq j \end{cases}. \quad (36)$$

In the case of no applied magnetic field, $s_{kl}^\mu = 0$, therefore $(\partial t_{kl}^\mu / \partial b_i) (\partial s_{kl}^\mu / \partial b_j)$ is reduced to zero. Introducing the effective reluctivity

$$\tilde{v}_{ij} = \left[\frac{\partial v_{ik} b_k}{\partial b_j} - \frac{\partial^2 t_{kl}^\mu}{\partial b_i \partial b_j} (s_{kl} - s_{kl}^\mu) + \frac{\partial t_{kl}^\mu}{\partial b_i} \frac{\partial s_{kl}^\mu}{\partial b_j} \right], \quad (37)$$

the constitutive law of magnetostrictive material can be expressed by the following system:

$$\begin{pmatrix} \tilde{\mathbf{T}} \\ \tilde{\mathbf{H}} \end{pmatrix} = \begin{bmatrix} \tilde{\mathfrak{c}} & -\tilde{\mathfrak{q}}^t \\ -\tilde{\mathfrak{q}} & \tilde{\mathfrak{v}} \end{bmatrix} \begin{pmatrix} \tilde{\mathbf{S}} \\ \tilde{\mathbf{B}} \end{pmatrix}. \quad (38)$$

The final local constitutive law of the system combined from Eqs. (38) and (23) is given by

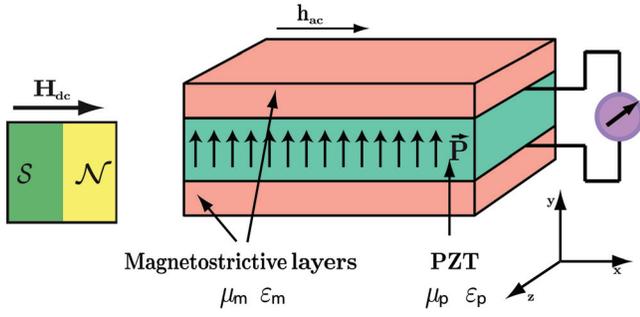


FIG. 1. (Color online) Magnetic sensor configuration.

$$\begin{pmatrix} \tilde{\mathbf{T}} \\ \tilde{\mathbf{D}} \\ \tilde{\mathbf{H}} \end{pmatrix} = \begin{bmatrix} \tilde{c} & \tilde{e}' & -\tilde{q}' \\ -\tilde{e} & \tilde{\epsilon} & 0 \\ -\tilde{q} & 0 & \tilde{\nu} \end{bmatrix} \begin{pmatrix} \tilde{\mathbf{S}} \\ \tilde{\mathbf{E}} \\ \tilde{\mathbf{B}} \end{pmatrix} \quad (39)$$

with $\tilde{e} = 0$ for the magnetostrictive material and $\tilde{q} = 0$ for the piezoelectric material.

V. APPLICATION—MAGNETIC SENSOR

A. Sensor configuration

The model has been applied to a magnetic sensor proposed by Huang Giang and Duc.¹⁵ In order to estimate the performance of such a sensor, we focus on the numerical modeling of the corresponding sandwiched structure presented in Fig. 1. The material parameters correspond to those of Terfenol-D (Appendix B) bonded with PZT (Appendix C). A schematic view of the sensor in 2D configuration is presented in Fig. 2.

The sensor is a trilayer consisting in a piezoelectric layer between two magnetostrictive layers. In order to enhance the sensitivity of the sensor for the measurement of a static magnetic field \mathbf{H}_{dc} , a low harmonic magnetic field \mathbf{h}_{ac} is usually superimposed using a coil surrounding the trilayer and excited at mechanical resonance frequency of the sensor ($\|\mathbf{h}_{ac}\| \ll \|\mathbf{H}_{dc}\|$). Therefore a harmonic electric voltage is obtained between the electrodes of the piezoelectric layer.

B. Modeling procedure

The numerical implementation of this magnetic sensor consists in two sequential finite element problems. The first is

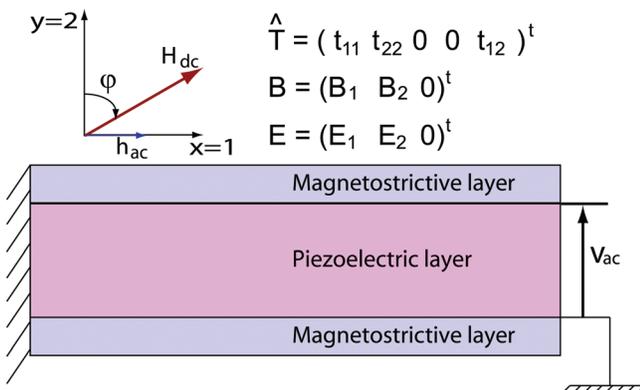


FIG. 2. (Color online) Magnetic sensor configuration.

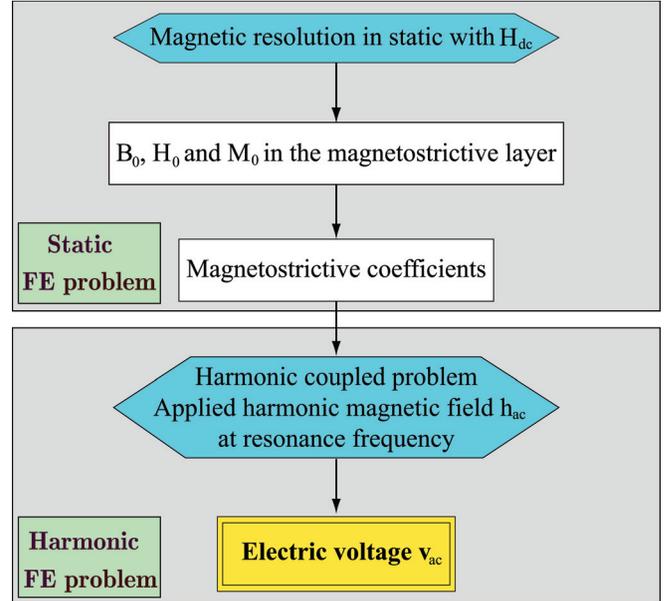


FIG. 3. (Color online) Modeling procedure.

a static problem allowing the calculation of the coefficients to establish the constitutive law of the magnetostrictive phase. The corresponding loading condition is no applied stress but initial magnetic induction due to the presence of the static magnetic field \mathbf{H}_{dc} . After obtaining all parameters of the magnetostrictive material under this specific loading, the second finite element problem is to solve the electric voltage under applied harmonic magnetic field at resonance frequency. The numerical procedure is detailed in Fig. 3.

C. Static FE problem

The magnetostrictive coefficients q_{ij} depend on the applied static magnetic field \mathbf{H}_{dc} . We first investigate with the proposed 2D model the value of these coefficients. Figures 4 and 5, respectively, plot the value of q_{11} and q_{21} as a function of the magnetic induction in the magnetostrictive layer. q_{11} (resp. q_{21}) links the component 11 (resp. 22) of the stress to the component 1 of the magnetic induction.

In the case of the sensor studied in this paper the magnetic induction in the sensor is redirected inside the magnetostrictive phase, whatever the orientation of the magnetic field outside the sensor. The induction is then mainly along direction 1 (in-plane). As shown in Figs. 4 and 5, the values of q_{11} and q_{21} are almost insensitive to the out-of-plane component when it is very low. Thus, the determination of the in-plane component of the induction will be sufficient to define the magnetostrictive parameters with good accuracy. Moreover, it can be shown that the in-plane component of the induction in the magnetostrictive layer is proportional to $\mathbf{H}_{eff} = \mathbf{H}_{dc} \sin \varphi$, where φ is defined on Fig. 6.

D. Harmonic FE problem

As the magnetostrictive material has a low conductivity, eddy currents have negligible effect at the resonance frequency,⁶ therefore the resonance frequency is the same as the mechanical resonance frequency: 73 kHz for the first

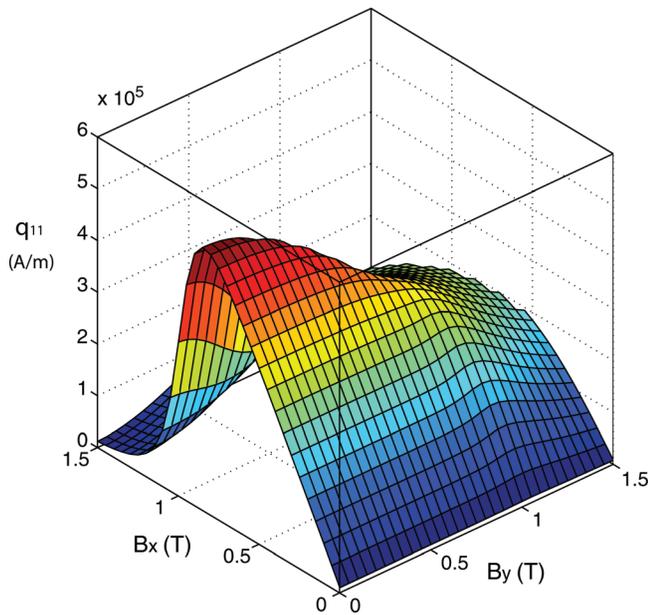


FIG. 4. (Color online) Magnetostrictive coefficient q_{11} as a function of the magnetic induction in the material.

longitudinal mode. The magnetostrictive parameter obtained with the static FE problem are implemented to solve the harmonic FE problem.

Figure 7 presents the electric voltage between the electrodes of the piezoelectric layer as a function of the applied in-plane static magnetic field. The electric voltage increases linearly in a first time and then decreases and approaches 0.

The numerical results have a shape very similar to the experimental results obtained by Huong Giang and Duc.¹⁵ This shape is highly correlated to the coefficient q_{11} . Indeed considering the sensor configuration, the harmonic magnetic field \mathbf{h}_{ac} in the magnetostrictive layer is only in the in-plane

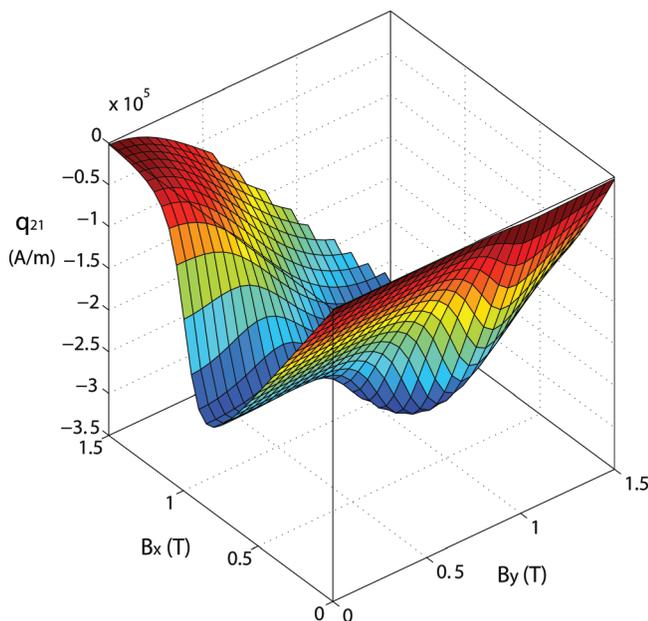


FIG. 5. (Color online) Magnetostrictive coefficient q_{21} as a function of the magnetic induction in the material.

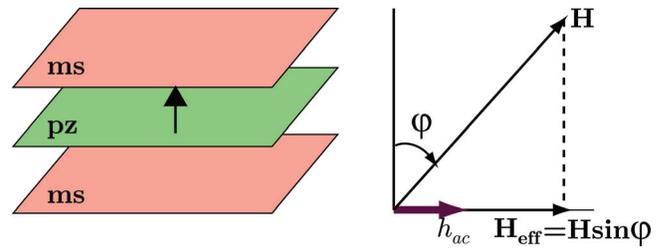


FIG. 6. (Color online) Rotation of the static field.

direction. In these conditions only the coefficients q_{11} and q_{21} play a role in the sensor response. The coefficient q_{21} links the in-plane component of the magnetic field to the strain along the direction normal to the layers. As the mechanical boundaries are free on the upper and lower borders of the sensor in the considered configuration, this strain along direction 2 will not be transmitted to the piezoelectric layer. Thus, for this particular sensor configuration and boundary conditions, the obtained electric voltage is mainly related to the coefficient q_{11} .

Moreover the obtained ME coefficients are much higher than those obtained in the static case. The static coefficients can be retrieved by moving the frequency of \mathbf{H}_{dc} far from the resonance frequency. This enhancement of ME coefficients⁷ by adding a small magnetic field oscillating at resonance frequency \mathbf{h}_{ac} is due to the nonlinearity of magnetic and magnetostrictive behavior. Indeed if this behavior was linear, a superimposition principle would apply, and only the electric response corresponding to the alternative component of the magnetic field would be amplified by the resonant effect. The response corresponding to the static magnetic field would only be amplified in an amount corresponding to the static magnetoelectric coefficient. As the behavior is non-linear, the static and resonant effects are coupled, and the magnetoelectric effect corresponding to the static field is enhanced through the resonance of the device.

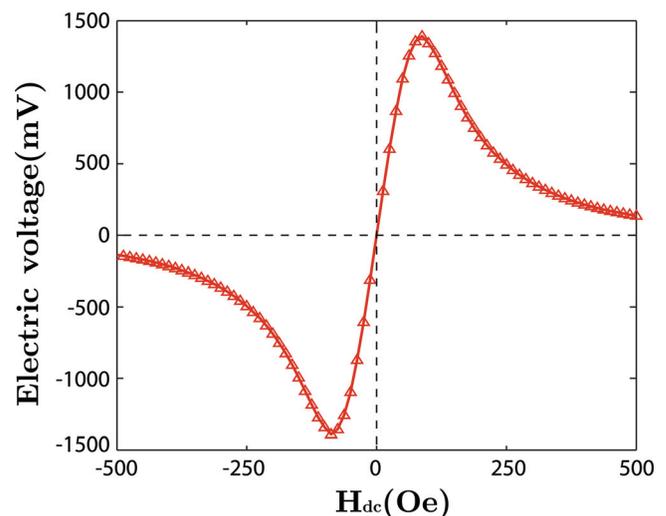


FIG. 7. (Color online) Electric voltage vs magnetic field applied in the in-plane direction.

E. Rotation of magnetic static field

The magnetic sensors can be used not only to measure the amplitude of static magnetic field but also its orientation. We consider the static magnetic field \mathbf{H}_{dc} turning around the harmonic magnetic field \mathbf{h}_{ac} , which is along the polarization's direction of the magnetostrictive layers. φ is the angle between \mathbf{H}_{dc} and the polarization's direction of the piezoelectric layer presented in Fig. 6. The rotation of \mathbf{H}_{dc} can be simulated by orthogonal projections of \mathbf{H}_{dc} onto x and y and writing the corresponding Neumann boundary conditions. Figure 8 (left hand side) shows the electric voltage as a function of the rotation angle. On the right-hand side, this voltage has been plotted as a function of the effective in-plane component $H_{dc} \sin \varphi$ of the applied magnetic field.

Again, there is a very satisfactory qualitative agreement with the experimental results of Huong Giang and

Duc.¹⁵ A quantitative comparison process should be undertaken, but the properties of the constituents have to be identified precisely. This is a work in progress. The right-hand side plots indicate that as far as the material remains in the linear stage of the magnetic behavior, the magneto-electric response is proportional to $\sin \varphi$. The magnetic saturation modifies this dependence to the orientation of the applied magnetic field. The right-hand side plots indicate that the magneto-electric response of the sensor when plotted as a function of $H_{dc} \sin \varphi$ is the same than the uniaxial characterization of Fig. 7. This result is consistent with the observation mentioned in the static analysis (Sec. V C) that—for this sensor configuration—the magnetic induction in the magnetostrictive layer is mainly directed in the in-plane direction and itself proportional to $H_{dc} \sin \varphi$. For this sensor configuration, a unique characterization with a static magnetic field in the in-plane direction is enough to define

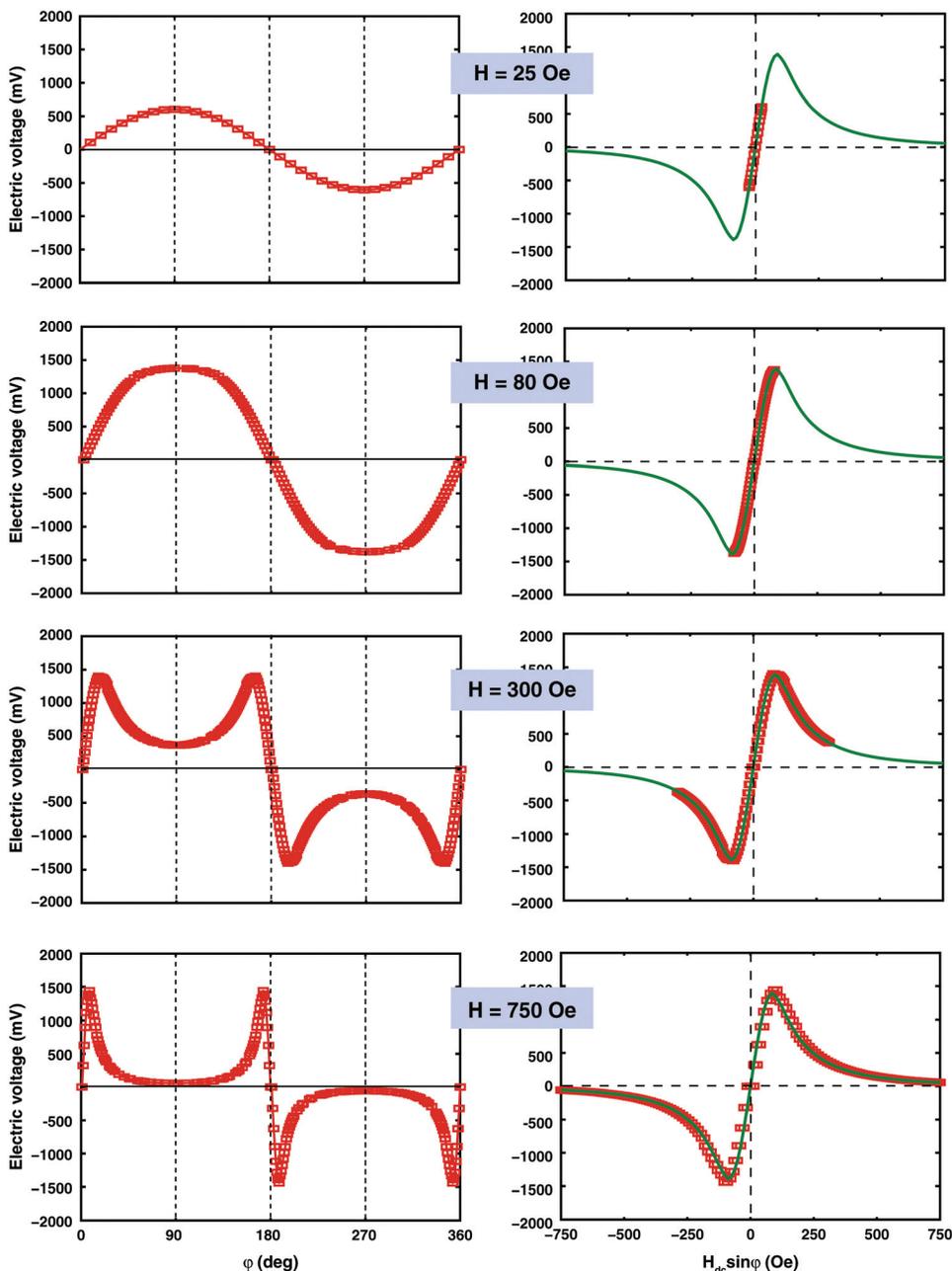


FIG. 8. (Color online) Electrical response of the ME sensor for different static magnetic-field values as a function of the rotation angle φ (left) and of the effective in-plane magnetic-field component $\mathbf{H}_{eff} = \mathbf{H}_{dc} \sin \varphi$ (right). The straight lines reproduce the result of Fig. 7 ($\varphi = 90^\circ$).

the response for a static magnetic field in any direction. This sensor cannot be used to measure the orientation and the intensity of the magnetic field in a single measurement. A 2D magnetic sensor could be built using two multilayer devices (or performing two measurements in different directions). It could also be interesting to change the mechanical boundary conditions of the sensor in order to let the coefficient q_{21} play a role in the overall response of the sensor. The choice of another type of microstructure (for instance, matrix/inclusions instead of multilayers) could also be studied. The proposed modeling approach provides a tool to explore such an optimization of 2D magnetic sensors based on magnetoelectric effect.

VI. CONCLUSION

In this paper, we propose a 2D finite element model to investigate the magnetoelectric effect under harmonic loadings for high sensitivity magnetic field sensors. In such piezoelectric/magnetostrictive composites, the piezoelectric material is prepolarized. The corresponding constitutive law is thus defined as linear. The magnetostrictive material is not prepolarized, and a nonlinear constitutive law has to be used. An appropriate linearization procedure depending on the polarization point of the magnetostrictive material is presented. The model has been applied to a typical configuration of magnetic sensor, with very satisfying qualitative results, whatever the relative orientation between the sensor and the applied magnetic field. This approach gives a deepened insight on the magnetoelectric sensor working principle. The enhancement of the magnetoelectric coefficient when a low amplitude harmonic field is superimposed to the static field to be measured is shown to be related to the nonlinearity of magnetic and magnetostrictive behavior. The proposed modeling provides a tool to explore the possibility to build magnetic sensors with optimal configurations—topology, boundary conditions—for high sensitivity 2D or 3D measurements. An experimental validation in order to perform quantitative comparison is a work in progress. The development of a 3D model is the further step of this study.

APPENDIX A: VOIGT NOTATION

Voigt notation takes advantage of the symmetry properties of a tensor to reduce its order. The stress tensor $\hat{\mathbf{T}}$, the strain tensor $\hat{\mathbf{S}}$ and the stiffness tensor $\hat{\mathbf{C}}$ are presented as follows:

$$\hat{\mathbf{T}} = (t_{11} \quad t_{22} \quad 0 \quad 0 \quad 0 \quad t_{12})^t,$$

$$\hat{\mathbf{S}} = (s_{11} \quad s_{22} \quad s_{33} \quad 2s_{23} \quad 2s_{31} \quad 2s_{12})^t,$$

$$\hat{\mathbf{C}} = \begin{pmatrix} c_{1111} & c_{1122} & c_{1133} & c_{1123} & c_{1131} & c_{1112} \\ c_{2211} & c_{2222} & c_{2233} & c_{2223} & c_{2231} & c_{2212} \\ c_{3311} & c_{3322} & c_{3333} & c_{3323} & c_{3331} & c_{3312} \\ c_{2311} & c_{2322} & c_{3333} & c_{2323} & c_{2331} & c_{2312} \\ c_{3111} & c_{3122} & c_{3133} & c_{3123} & c_{3131} & c_{3112} \\ c_{1211} & c_{1222} & c_{1233} & c_{1223} & c_{1231} & c_{1212} \end{pmatrix}.$$

APPENDIX B: PROPERTIES OF MAGNETOSTRICTIVE MATERIAL

(1) Stiffness tensor:

$$\begin{bmatrix} 1.24 & 0.61 & 0.61 & 0 & 0 & 0 \\ 0.61 & 1.42 & 0.61 & 0 & 0 & 0 \\ 0.61 & 0.61 & 1.42 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.54 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.54 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.63 \end{bmatrix} \times 10^{10} \text{Pa};$$

- (2) magnetic properties: $\mu_0 M_s = 1T$, $\chi_0 = 99$;
 (3) magnetostrictive parameter: $\beta = 25 \times 10^{-6}$;
 (4) density⁴: 9200 kg/m³.

APPENDIX C: PROPERTIES OF PIEZOELECTRIC MATERIAL

(i) Stiffness tensor:

$$\begin{bmatrix} 3.19 & 1.43 & 1.43 & 0 & 0 & 0 \\ 1.43 & 3.19 & 1.43 & 0 & 0 & 0 \\ 1.43 & 1.43 & 2.67 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.68 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.58 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.58 \end{bmatrix} \times 10^{10} \text{Pa};$$

(ii) Dielectric permittivity⁴:

$$\begin{bmatrix} 15.92 & 0 & 0 \\ 0 & 15.92 & 0 \\ 0 & 0 & 15.92 \end{bmatrix} \times 10^{-9} \text{A(s/V m)};$$

(iii) Piezoelectric matrix⁴:

$$\begin{bmatrix} 0 & -5.9 & 0 \\ 0 & -5.9 & 0 \\ 0 & 15.2 & 0 \\ 0 & 0 & 10.5 \\ 0 & 0 & 0 \\ 10.5 & 0 & 0 \end{bmatrix} \times \text{N}/(\text{V m)};$$

(iv) Density⁴: 7700 kg/m³.

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