3-D Semi-Analytical Homogenization Model for Soft Magnetic Composites

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A 3-D semi-analytical homogenization model for soft magnetic composites (SMCs) is presented. The model is able to handle the nonlinear magnetic behavior of the ferromagnetic particles and estimate the eddy current (EC) losses. Comparisons to finite element (FE) results show a very good agreement for a unit cell of composite. The model can be used in a structural FE study of electromagnetic devices made of SMCs.

***Index Terms*—Eddy current (EC) losses, effective permeability, Maxwell-Garnett estimate, nonlinear behavior, soft magnetic composite (SMC).**

I. Introduction

SOFT magnetic composites (SMCs) exhibit low levels of eddy current (EC) losses when subjected to magnetic loadings [1], particularly when compared to bulk or laminated ferromagnetic materials. The loss reduction is due to the insulation of magnetic particles in a dielectric polymer matrix. In order to maintain good magnetic performance, the volume fraction of magnetic particles needs to be high, typically around 95%.

Meshing the microstructure of SMC during the numerical design of magnetic devices using such materials is not feasible because of the huge difference in scales. That is the reason why homogenization models providing the macroscopic magnetic behavior [2], [3] and the macroscopic EC loss estimate [4], [5] can be useful.

Many models are based on numerical approaches (such as finite element (FE) models), providing a very accurate description of magnetic field [6] and EC loss distribution [7]. But these models usually lack predictivity, meaning that a change in one of the parameters requires a recomputation from scratch.

Another approach is based on (semi-)analytical models of the macroscopic magnetic behavior and the macroscopic EC loss estimate. These models [8] are often based on simplifying assumptions (such as linear magnetic behavior and 2-D model) which make them useful only in limited cases.

A recent article [9] introducing (quasi-)analytical models for 2-D SMC shows that the EC loss density can be accurately estimated even with nonlinear magnetization for the magnetic particles. This approach brings more predictivity since it is based on an analytical formula; however, these models are dealing with simple EC distribution (the current amplitude in the 2-D particles was varying linearly with space), which is not the case in SMC in general, especially because of their 3-D nature.

In this article, periodic high concentration SMC with cube-shaped inclusions exhibiting nonlinear magnetic behavior is studied. Such a microstructure leads to complex current distribution in the magnetic particles and a semi-analytical homogenization model of EC loss density is derived. The results are compared to the ones obtained from an FE model and an application on a 3-D magnetic circuit made of SMC is presented.

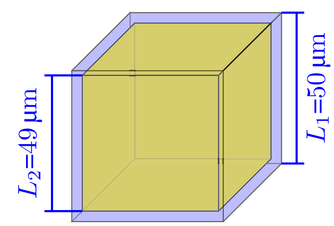

Fig. 1. 3-D unit cell of SMC: a cubic particle of ferromagnetic material (in yellow) is perfectly insulated electrically by a dielectric layer (in blue).

II. Problem Definition

A. Microstructure/Parameters

The SMC microstructure is simplified in this study to a 3-D periodic array of cube-shaped inclusions (iron, index 2) in a dielectric matrix (epoxy, index 1) in order to simplify the FE study for comparing the results. Fig. 1 shows the simplified microstructure of SMC for this study.

However, the method should still be valid for more generic microstructures, as it was demonstrated to be the case for SMC made of particles exhibiting linear magnetic behavior [10].

The magnetization curve for iron particles is shown in Fig. 2. The electric conductivity σ_2 of iron particles

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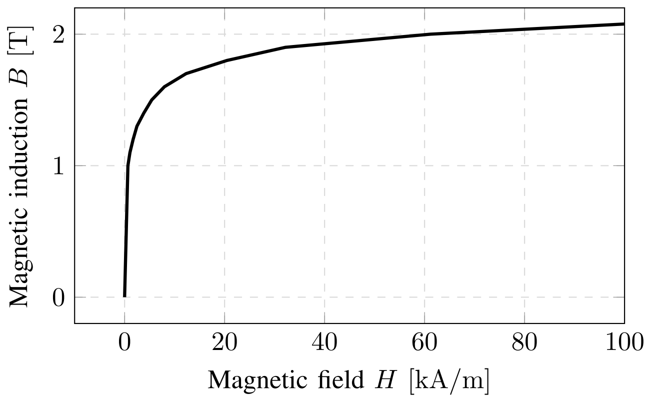

Fig. 2. Magnetization curve of iron [11].

is 1.12×10^7 S/m. Epoxy is modeled by a null conductivity and a permeability μ_1 equal to the vacuum permeability μ_0.

With the geometric parameters shown in Fig. 1, iron particles exhibit a volume fraction ξ_2 of 94.12% in the SMC.

B. Key Assumption

Similar to the assumption used in the model for 2-D SMC in a simple configuration [9], the models for 3-D SMC will also rely on the assumption of a uniform magnetic induction $\mathbf{B}_2$ (or a uniform magnetic field $\mathbf{H}_2$) in iron particles when a macroscopic induction $\overline{\mathbf{B}}$ (or macroscopic magnetic field $\overline{\mathbf{H}}$) is applied to the cell (barred quantities represent the macroscopic values, which are volume averages over the whole unit cell). This assumption is reasonable when the composite exhibits a very high concentration of ferromagnetic particles.

This assumption will be verified in Section II-C through a FE model of a unit cell of SMC.

C. Magnetic Behavior

Assuming that the EC loss level is relatively low in the iron particles, a quasistatic homogenization approach will be used for the prediction of the macroscopic magnetic behavior of SMC. The problem can be turned into an iterative homogenization model using linear homogenization tools since the magnetic field is uniform in iron particles: the whole iron behavior will be described by a single secant permeability value μ_2^s, and the secant effective permeability $\widetilde{\mu}_{\mathrm{MG}}^s$ will be a classical 3-D Maxwell-Garnett model [12] (equivalent to Ollendorf's model [13]).

The different steps of the iterative approach are similar to the ones in Fig. 4 in Ref. [9] except that the 3-D secant effective permeability is now defined by

$$\widetilde{\mu}_{\mathrm{MG}}^s = \frac{(\mu_2^s + 2\mu_1) + 2(\mu_2^s - \mu_1)\xi_2}{(\mu_2^s + 2\mu_1) - (\mu_2^s - \mu_1)\xi_2}\mu_1. \tag{1}$$

The convergence criterion used in the nonlinear iterative scheme is

$$\frac{||\mathbf{H}_2^i - \mathbf{H}_2^{i-1}||}{||\overline{\mathbf{H}}||} < 0.1\% \tag{2}$$

where the superscript i indicates the iteration number and $||.||$ is the Euclidean norm.

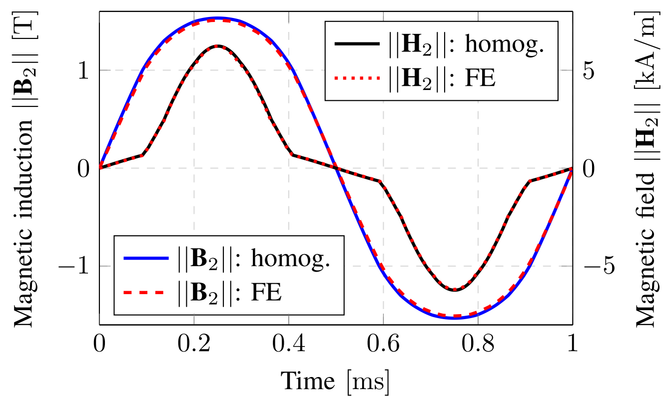

Fig. 3. Time evolution of magnetic field $||\mathbf{H}_2||$ and magnetic induction $||\mathbf{B}_2||$ evaluated at the center of the iron particle when a 1 kHz sinusoidal macroscopic magnetic field $\overline{\mathbf{H}}$ (magnitude: 30 kA/m) is applied to the unit cell.

In order to validate the uniform magnetic induction assumption for the iron particle, a time-dependent FE model of a 3-D unit cell, such as the one on Fig. 1 is built, and a sinusoidal macroscopic magnetic field $\overline{\mathbf{H}}$ is applied. The application of a macroscopic magnetic field instead of a macroscopic magnetic induction is obtained through a constraint (in COMSOL Multiphysics, with nonlinear iterative determination) on the potential vector circulating on four faces of the unit cell.

Fig. 3 shows the time evolution of the magnetic field $\mathbf{H}_2$ and the magnetic induction $\mathbf{B}_2$ at the center of the cubic particle when the sinusoidal macroscopic magnetic field $\overline{\mathbf{H}}$ has a frequency of 1 kHz. The values of $\mathbf{H}_2$ and $\mathbf{B}_2$ predicted by quasi-static homogenization tools are also shown.

It can be seen that the results are in very good accordance. However, the homogenization model slightly overestimates the magnetic induction in the iron particle. Such a statement was also observed for 2-D SMC [9] and can be explained by the opposing field generated by EC, which is taken into account in the FE model but not in the quasi-static homogenization approach here.

The variance V_2 of magnetic induction $\mathbf{B}$ in the iron particle is also postprocessed in the FE model

$$V_2 = \frac{\langle \mathbf{B}(\mathbf{x}) \cdot \mathbf{B}(\mathbf{x})\rangle_2 - \langle \mathbf{B}(\mathbf{x})\rangle_2 \cdot \langle \mathbf{B}(\mathbf{x})\rangle_2}{\langle \mathbf{B}(\mathbf{x})\rangle_2 \cdot \langle \mathbf{B}(\mathbf{x})\rangle_2} \tag{3}$$

with $\langle . \rangle_2$ the volume average operator over the iron particle only. It has been observed that the variance V_2 in the FE model is less than 0.2% in general which confirms that the assumption of uniform magnetic induction $\mathbf{B}_2$ in the particle is valid. For higher frequencies, this assumption may fail with the appearance of skin effect (as shown in Ref. [9] and with the same parameters, the assumption remains valid up to 10 kHz).

Fig. 4 shows the magnetization response of the unit cell determined by FE which was obtained by postprocessing the peak value of the macroscopic induction $\overline{\mathbf{B}}$ for multiple sinusoidal macroscopic magnetic field $\overline{\mathbf{H}}(t)$ with different magnitudes (frequency: 1 kHz).

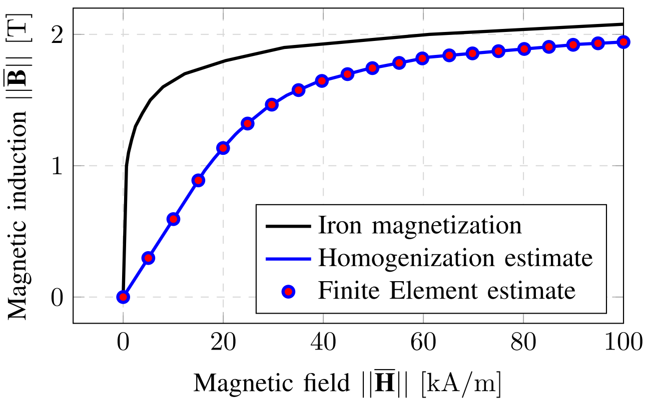

Fig. 4. Macroscopic magnetization of a cubic cell of SMC.

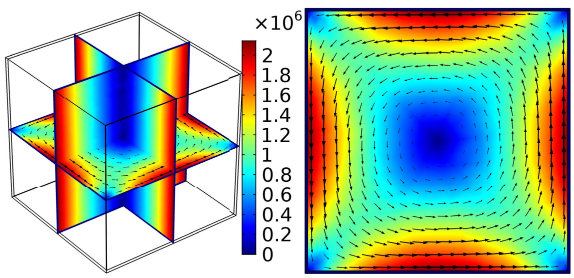

Fig. 5. FE result: EC distribution (unit: A/m^2) in slices of the unit cell at $t = 0.5$ ms (see Fig. 3, frequency: 1 kHz).

The results are again in very good accordance because the assumptions used for developing the model are verified (uniformity of magnetic field in the particle and no skin effect).

D. EC Losses

The EC loss density $\mathcal{U}$ is defined as the Joule losses dissipated per unit volume during a wave period T

$$\mathcal{U} = \left\langle \int_0^T \sigma \mathbf{E}^2 dt \right\rangle \tag{4}$$

where σ is the electric conductivity, $\mathbf{E}$ the electric field, and the operator $\langle . \rangle$ is the volume average.

In the case of EC loss prediction for 2-D SMC [9], the EC distribution was varying linearly with regard to space within the iron particle, which made it simple to integrate for the prediction of EC loss density [see (4)]. But in this 3-D case, the EC distribution is much more complex as shown in Fig. 5.

From Maxwell–Faraday equation, one can deduce that the magnitude of the EC in the particle is directly proportional to $\partial \mathbf{B}_2/\partial t$. It implies that the EC loss density $\mathcal{U}_{\text{homog}}$ for a unit cell, computed from (4), can be simplified as [8]

$$\mathcal{U}_{\text{homog}} = \beta \int_0^T \left(\frac{d\mathbf{B}_2}{dt} \right)^2 dt \tag{5}$$

with β a coefficient depending on the shape and size of the particle and its electrical conductivity σ_2. The determination of β can be done numerically (with FE for example), but in that particular case of a cubic inclusion, the integral of series

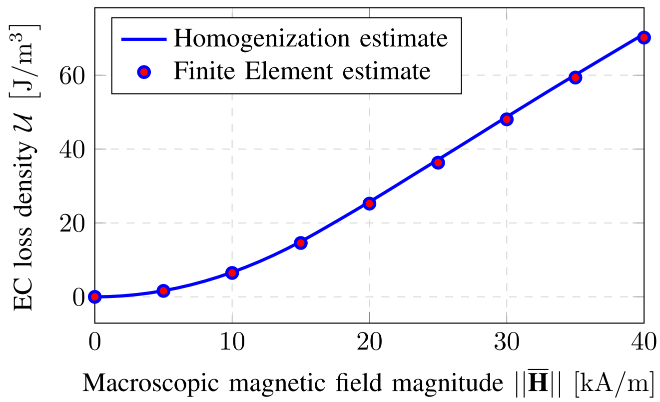

Fig. 6. EC losses density of a cubic cell of SMC as a function of macroscopic magnetic field magnitude $||\overline{\mathbf{H}}||$. Frequency: 1 kHz.

expansion can provide a formula [8]

$$\mathcal{U}_{\text{homog}} = \frac{9\xi_2\sigma_2 L_2^2}{64} \int_0^T \left(\frac{d\mathbf{B}_2}{dt} \right)^2 dt \tag{6}$$

with L_2 the size of cubic particles, σ_2 the electric conductivity of ferromagnetic inclusions, and ξ_2 the filling factor of inclusions (94.12% in this study).

Fig. 6 shows the EC loss density of a cubic cell of SMC determined by FE when a sinusoidal macroscopic magnetic field (magnitude: $||\overline{\mathbf{H}}||$, frequency: 1 kHz) is applied.

The homogenization results are obtained through the use of quasi-static homogenization tools localizing the magnetic induction $\mathbf{B}_2(t)$ in the iron particle as a function of the sinusoidal macroscopic magnetic field $\overline{\mathbf{H}}(t)$, and the time derivative and integration in (6) are performed numerically.

These results show that the EC loss density estimated by homogenization is very close to the one predicted by the FE model. The difference is less than 2% in this study. However, it can be noticed that the EC loss density determined by homogenization is always overestimating the one obtained by FE. It is again due to the fact that the homogenization approach slightly overestimates the magnetic induction in the iron particle (see Fig. 3), hence overestimating the EC losses.

III. Use of Homogenization Model

An application of the homogenization model used in a 3-D time-dependent FE study of a magnetic circuit with SMC as the magnetic core is developed in this section. The geometry of the magnetic circuit is shown in Fig. 7.

Only one-fourth of the domain (including the surrounding air box) was modeled by taking advantage of the symmetries. A sinusoidal current at 1 kHz is imposed in the coil with a magnetomotive force of 10 kA-turns. The homogenized nonlinear magnetization used for the magnetic core is the one obtained by homogenization (as shown in Fig. 4), and the electrical conductivity is set to zero.

The domain is meshed with tetrahedrons (maximum size in the magnetic core: 4 mm), leading to a numerical system with close to 400 000 degrees of freedom.

This lossless model is expected to still give an accurate description of the magnetic field distribution as it was demonstrated in the case of 2-D SMC [9] because the EC losses in

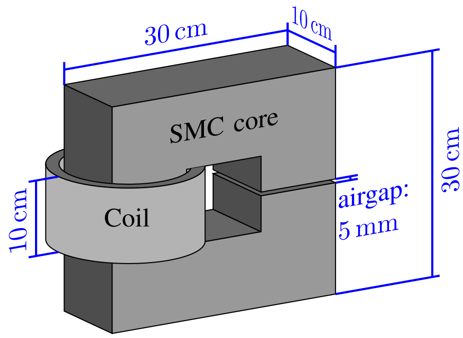

Fig. 7. Geometry of the magnetic circuit made of SMC with its coil (inner radius: 7.5 cm, outer radius: 9 cm) for the 3-D FE study.

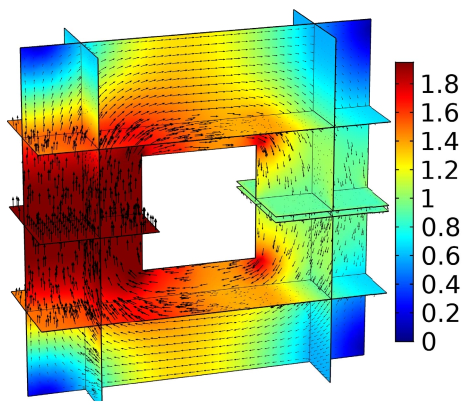

Fig. 8. Magnetic induction **B** distribution (unit: T) in different slices of the magnetic core when the current in the coil is at peak value.

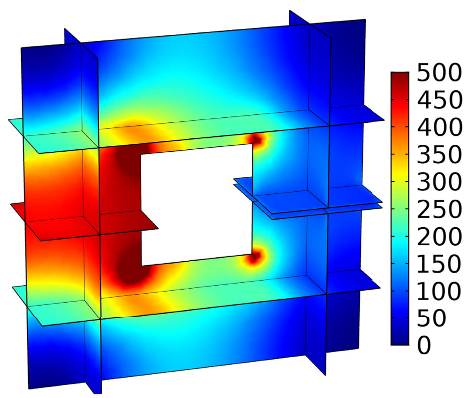

Fig. 9. EC loss density $\mathcal{U}$ (unit: J/m^3) in different slices of the magnetic core. Frequency: 1 kHz.

SMC are assumed to be small enough to not modify significantly the magnetic field distribution. The EC loss density $\mathcal{U}$ can then be postprocessed by time derivative and integration of the magnetic induction in the magnetic core using (6) (with a localization rule to recompute $\mathbf{B}_2$ as a function of the local macroscopic field $\overline{\mathbf{H}}$ or the macroscopic induction $\overline{\mathbf{B}}$).

Fig. 8 presents the magnetic induction distribution in different slices of the study domain when the current is maximum in the coil. It can be seen that the magnetic core around the coil is highly saturated magnetically, as well as the inside corner regions.

Fig. 9 shows the EC loss density $\mathcal{U}$ predicted by homogenization in different slices of the magnetic core. A volume integration over the magnetic core would also give the user the information of energy loss due to EC during one period of the loading.

The accuracy of EC loss prediction with this homogenization approach cannot be estimated since it would require to build an equivalent FE model taking into account every iron particle in the magnetic circuit, which is impossible in 3-D. Such validation was made in the simple 2-D SMC application [9], and the EC loss prediction was overestimated by 4.5%. It is anticipated that a similar accuracy is obtained in this 3-D case since the two studies on the unit cell (2-D and 3-D) gave a very similar accuracy.

IV. Conclusion

A homogenization model was developed to handle nonlinear magnetic behavior for estimating the magnetic behavior and the EC loss density in SMC with 3-D microstructures. The model is analytical and is an extension of a model developed for simple 2-D SMC structures [9]. The novelty of the model is that it can deal with complex EC distribution in the iron particles such as the one exhibited in a cubic particle, which is the case in general because SMC exhibits 3-D microstructure. The comparison with an FE model of a unit cell of SMC shows that the homogenization model exhibits a good accuracy for both the magnetic behavior and the estimation of EC loss density.

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