Optimal Design of Magnetostrictive Composites: An Analytical Approach

Romain Corcolle, Laurent Daniel, and Frédéric Bouillault

Laboratoire de Génie Electrique de Paris, CNRS UMR 8507; SUPELEC; Univ Paris-Sud; Univ Pierre et Marie Curie, Plateau de Moulon, 91192 Gif sur Yvette Cedex, France

Giant magnetostriction materials, such as Terfenol-D, have allowed the development of a new class of actuators and sensors based on magnetoelastic properties. However, their mechanical properties are a limiting factor for some applications. Composites, made of Terfenol-D particles in a matrix, can bypass these limitations, improving the overall mechanical properties while maintaining the magnetostriction effect. But, to optimize the design of magnetostrictive composites, advanced modeling tools are needed. This paper proposes a homogenization model based on inclusion problems. It compares modeling results to experimental data from the literature, and it illustrates the use of the model as a tool for optimal design.

Index Terms—**Composite, effective properties, homogenization, inclusion, magnetostriction, Terfenol-D.**

I. Introduction

FOR a number of years, magnetostrictive composites have received much attention through sensor or actuator studies. Because of its giant magnetostriction, Terfenol-D particularly fits such uses. But applications are limited by its cost and brittleness. In order to improve mechanical properties, the use of a matrix with Terfenol-D particles gives a magnetostrictive composite with still intermediate magnetostriction [1]. These materials are well suited to high-frequency applications. In that context, there is a need for modeling tools to link the effective behavior of these composites to their composition. Many factors influence the effective properties among which volumetric fraction of Terfenol-D and mechanical properties of the matrix are very sensitive. Modeling tools can lead to the definition of optimal composition for such composites, adapted to the requirements for the final actuator or sensor.

A possibility would be the use of finite-element modeling to predict composites behavior. The limitation of such an approach is that a finite-element calculation requires the complete knowledge of the microstructure and is valid only for that particular microstructure. In that paper, we propose a semianalytical model for the prediction of the effective behavior of composites, using some basic and limited statistical information about the microstructure. Such strategies based on homogenization theory have been developed over recent years.

Herbst *et al.* developed a single sphere model [2]. This model is limited to only one single magnetostrictive phase and appeared to be nearly insensitive to material constants [3], [4]. Zhou *et al.* developed another model [5] but still limited to one single magnetostrictive phase and nonapplicable for nonlinear magnetic behaviors. Nan *et al.* proposed a more rigorous, but also more complicated, model based on Green's functions calculation [3], [4]. Another model has been proposed by Feng *et al.*, based on the double-inclusion method [6] but the improvement compared to the simple inclusion one seems to be weak.

The model presented in this paper is based on basic inclusion problems. Its use is not limited to one single magnetostrictive phase but can be applied to n-phase composites. Moreover, nonlinear magnetic behavior of magnetostrictive phases can be taken into account. The magnetostriction strain of Terfenol-D particles is deduced from the macroscopic applied magnetic field. From the magnetostriction strain of Terfenol-D, macroscopic strain/stress of the composite can be deduced analytically. In the first part, the principle of the homogenization model is presented. The application to a biphasic composite is detailed in the second part. The results are compared to experimental data from the literature. The third part is dedicated to the use of the model as a tool for optimal design of magnetostrictive composites.

II. Homogenization

A. Principle

Homogenization purpose is the determination of effective properties of heterogeneous materials. The objective is to deduce the material properties of a fictive homogeneous material equivalent to the real heterogeneous one from the properties of its constituents and some assumptions about the microstructure.

We consider an n-phasic material with phases $\{1, 2, \ldots, n\}$. The material properties of phase i are the magnetic permeability second-order tensor $\underline{\underline{\mu}}^i$ and the elastic stiffness fourth-order tensor $\mathbb{C}^i$. The mean magnetic induction, magnetic field, stress, and strain tensors in phase i are noted respectively $\mathbf{B}^i$, $\mathbf{H}^i$, $\boldsymbol{\sigma}^i$, $\boldsymbol{\varepsilon}^i$. The magnetic and mechanical constitutive laws are written (using Einstein's summation convention) according to (1) and (2)

$$\mathbf{B}^i = \underline{\underline{\mu}}^i \mathbf{H}^i \quad \text{i.e.,} \quad B^i_k = \mu^i_{kl} H^i_l \tag{1}$$

$$\boldsymbol{\sigma}^i = \mathbb{C}^i : \boldsymbol{\varepsilon}^i \quad \text{i.e.,} \quad \sigma^i_{kl} = C^i_{klmn} \varepsilon^i_{mn}. \tag{2}$$

The effective permeability tensor $\widetilde{\underline{\underline{\mu}}}$ of the composite is defined according to (3)[1]

$$\overline{\mathbf{B}} = \widetilde{\underline{\underline{\mu}}}\,\overline{\mathbf{H}} \tag{3}$$

with $\overline{\mathbf{B}} = \langle \mathbf{B^i} \rangle$ and $\overline{\mathbf{H}} = \langle \mathbf{H^i} \rangle$.

In a similar way, the effective stiffness tensor of the composite $\widetilde{\mathbb{C}}$ is defined according to (4):

$$\overline{\boldsymbol{\sigma}} = \widetilde{\mathbb{C}} : \overline{\boldsymbol{\varepsilon}} \tag{4}$$

with $\overline{\boldsymbol{\sigma}} = \langle \boldsymbol{\sigma}^i \rangle$ and $\overline{\boldsymbol{\varepsilon}} = \langle \boldsymbol{\varepsilon}^i \rangle$.

[1] $\langle . \rangle$ denotes an averaging operation over the volume.

Digital Object Identifier 10.1109/TMAG.2007.910551

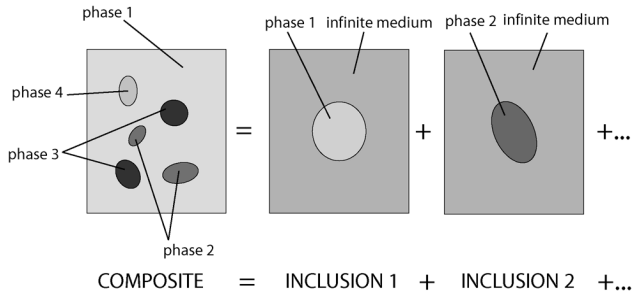

Fig. 1. Principle of inclusion based models.

The basic inclusion problem allows the definition of many homogenization models [7], both in mechanics and magnetics.

B. Homogenization Models Based on Inclusion Problems

Inclusion based models rely on the hypothesis that mean fields in each phase i are similar to corresponding fields of an inclusion of phase i embedded in an infinite homogeneous medium with magnetic property $\underline{\underline{\mu}}^m$ or mechanical property $\mathbb{C}^m$ (Fig. 1). The infinite medium is usually taken isotropic, both from a magnetic [8] and a mechanical [7] point of view

$$\mu_{ij}^m = \mu^m \delta_{ij} \tag{5}$$

$$\mathbb{C}^m = 3k^m \mathbb{J} + 2G^m \mathbb{K} = \{3k^m, 2G^m\} \tag{6}$$

with μ^m, k^m, and G^m respectively the magnetic permeability, bulk modulus, and shear modulus of the infinite medium; δ_{ij} the Kronecker symbol; $\mathbb{J}$ and $\mathbb{K}$ are normed basis tensors: $J_{ijkl} = (1/3)\delta_{ij}\delta_{kl}$, $K_{ijkl} = (1/2)(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) - (1/3)\delta_{ij}\delta_{kl}$. When the phase distribution in the composite is isotropic, the shape of the inclusion in the basic inclusion problem is spherical. It has to be noticed that the shape of the inclusion is related to the phase distribution and by no mean to the shape of the particles in the real composite.

The n-phasic material state can be deduced from the solution of n inclusion problems. This is the reason why the knowledge of the solution to the basic inclusion problem is required.

C. Basic Inclusion Problem

The basic inclusion problem is defined with:

- an inclusion embedded in an infinite medium;
- a uniform applied loading $\mathbf{H}^\infty$ or $\boldsymbol{\varepsilon}^\infty$ at the infinity.

Then, in the case of linear behavior for both media (inclusion i and infinite medium m) and an ellipsoidal shape for the inclusion, the field in the inclusion is uniform. This is the reason why homogenization models based on inclusion problems fit well with mean field homogenization theory.

For example, in the case of a spherical inclusion and isotropic linear behavior,[2] the magnetic field in the inclusion can be given analytically [9], [10]

$$\mathbf{H}^i = \frac{3}{2 + \frac{\mu^i}{\mu^m}} \mathbf{H}^\infty. \tag{7}$$

[2]For an ellipsoidal shape and anisotropic linear behavior, details can be found in [9].

Similarly, the strain in the inclusion can be deduced [7]

$$\boldsymbol{\varepsilon}^i = (\mathbb{C}^* + \mathbb{C}^i)^{-1} : (\mathbb{C}^* + \mathbb{C}^m) : \boldsymbol{\varepsilon}^\infty \tag{8}$$

with $\mathbb{C}^*$ the Hill constraint tensor (representing the influence of the infinite medium on inclusion strain), as a function of the stiffness tensors and the shape of the inclusion. In the case of a spherical inclusion, $\mathbb{C}^*$ only depends on the properties of the infinite medium

$$\begin{aligned} \mathbb{C}^* &= 4G^m \mathbb{J} + \frac{G^m}{3} \frac{9k^m + 8G^m}{k^m + 2G^m} \mathbb{K} \\ &= 3k^* \mathbb{J} + 2G^* \mathbb{K} \\ &= \{3k^*, 2G^*\}. \end{aligned} \tag{9}$$

Basic inclusion problems can be solved with (7) and (8). But, the definition of the n inclusion problems from the n-phase composite have to be detailed.

D. Choice of Infinite Medium Loading and Properties

The applied loading in each inclusion problem is usually different from the macroscopic applied loading on the real composite. Indeed it must be verified that the mean magnetic field (resp. strain) over the inclusions is equal to the mean magnetic field $\overline{\mathbf{H}}$ (resp. strain $\overline{\boldsymbol{\varepsilon}}$) in the composite.

Then, the applied loading can be different for each inclusion problem but the same applied loading ($\mathbf{H}^\infty$ and $\boldsymbol{\varepsilon}^\infty$) for every inclusion problems is often chosen for simplicity reasons.

The last thing to specify is the choice of the infinite medium properties. From this choice, the field estimates will vary considerably. But some specific choices enable the recovery of classical estimates in linear homogenization of biphasic materials such as Wiener bounds, Hashin & Shtrikman bounds, and self-consistent estimate [9].

The next section proposes an application of the proposed approach in the case of a biphasic composite.

III. Magnetostrictive Biphasic Composite

The magnetostrictive biphasic composite consists of magnetostrictive particles embedded in a nonmagnetostrictive matrix. In this paper, we will only consider isotropic behavior (meaning that material constants are the three following scalars: μ is the permeability, k is the bulk modulus, and G is the shear modulus) for both phases. The description of the microstructure is limited to the following information: volumetric fractions and isotropic distribution of magnetostrictive particles. The magnetostrictive volumetric fraction is f (and so the matrix one is $f' = 1 - f$).

The magnetostrictive effect only takes place in magnetostrictive particles and the magnetostriction strain is assumed, as a first approximation, to be stress independent. This latter assumption is not general—the magnetostriction strain is known to strongly depend on stress (see for instance [11]). But, in the particular case of Terfenol-D, beyond a stress value (a few MPa), Terfenol-D magnetostriction is nearly stress-independent when plotted as a function of the magnetic induction (see for instance [12]). This is the reason why this hypothesis is often considered.

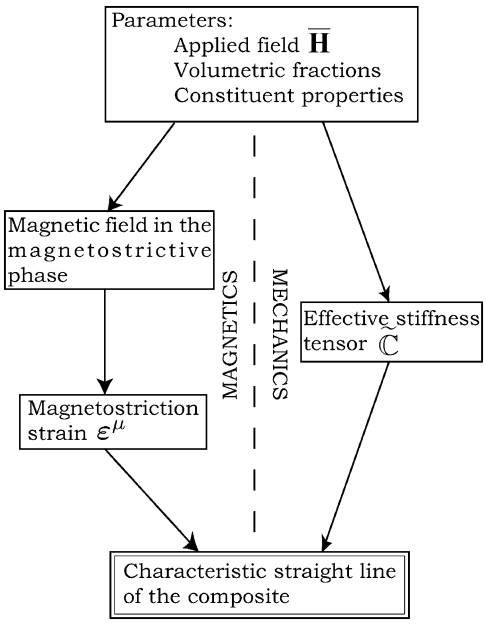

Fig. 2. Solving scheme for the proposed model.

The objective of the model is to link the macroscopic mechanical response to the applied macroscopic magnetic field. The solving scheme is presented in Fig. 2.

A. Magnetic Calculation

The first objective is to determine the mean magnetic field in magnetostrictive particles. For that purpose, the composite is reinterpreted in terms of two inclusion problems:

- a sphere representing the magnetostrictive phase ($\mathbf{ms}$) behavior embedded in an infinite medium;
- a sphere representing the matrix ($\mathbf{mat}$) behavior embedded in the same infinite medium.

The infinite medium properties are chosen in order to compute one Hashin & Shtrikman estimate: the infinite medium permeability is equal to the matrix one μ^{mat}.

The first inclusion problem equation is similar to (7) if linear magnetic behavior is considered:

$$\mathbf{H}^{\mathbf{ms}} = \frac{3}{2 + \frac{\mu^{\mathbf{ms}}}{\mu^{\mathbf{mat}}}} \mathbf{H}^{\infty}. \tag{10}$$

The second inclusion problem is homogeneous, leading to the following equation:

$$\mathbf{H}^{\mathbf{mat}} = \mathbf{H}^{\infty}. \tag{11}$$

The following condition must be verified, $\langle \mathbf{H}^{i} \rangle = \overline{\mathbf{H}}$, so that

$$f\mathbf{H}^{\mathbf{ms}} + (1-f)\mathbf{H}^{\mathbf{mat}} = \overline{\mathbf{H}}. \tag{12}$$

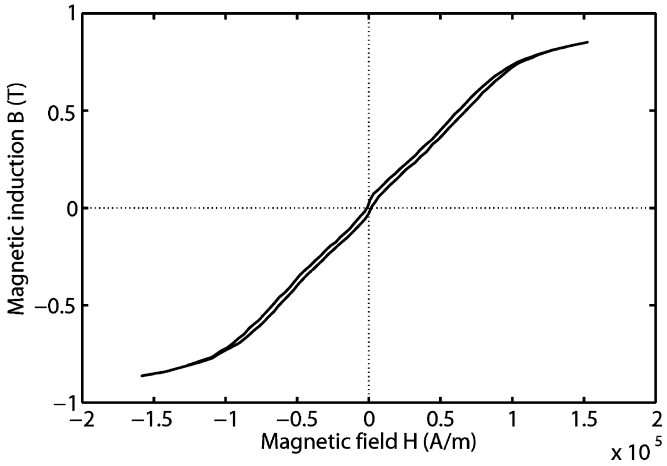

Fig. 3. Experimental magnetization curve for Terfenol-D [13] (no applied stress).

Developing the last three equations gives the mean magnetic field of the magnetostrictive particles $\mathbf{H}^{\mathbf{ms}}$, as a function of the macroscopic applied magnetic field $\overline{\mathbf{H}}$

$$\mathbf{H}^{\mathbf{ms}} = \frac{3}{2 + f + (1-f)\frac{\mu^{\mathbf{ms}}}{\mu^{\mathbf{mat}}}} \overline{\mathbf{H}}. \tag{13}$$

This last equation is true for linear magnetic properties. Unfortunately, magnetostrictive materials usually have a strongly nonlinear and hysteretic magnetic behavior. Fig. 3 presents an experimental magnetization curve for Terfenol-D [13].

Nonlinearity is usually complex to solve. But the presented method enables the computation of the mean magnetic field in the magnetostrictive particles with no further complexity. Indeed in the inclusion/matrix problem, the magnetic field is still uniform in the inclusion, even if the inclusion behavior is nonlinear. The sole difficulty is to solve the following equation (with nonconstant permeability):

$$\frac{1}{3}\left(2 + f + (1-f)\frac{\mu^{\mathbf{ms}}(\mathbf{H}^{\mathbf{ms}})}{\mu^{\mathbf{mat}}}\right)\mathbf{H}^{\mathbf{ms}} = \overline{\mathbf{H}}. \tag{14}$$

The mean magnetic field of magnetostrictive particles can be computed with an iterative scheme.

The second objective is to determine the mean magnetostriction strain of magnetostrictive particles from the previous local mean magnetic field. The magnetostriction strain is usually considered isovolumetric [14]. In such conditions, the magnetostriction strain tensor for isotropic materials (such as Terfenol-D) is given by (15) (assuming that the macroscopic applied magnetic field is along axis 1)

$$\boldsymbol{\varepsilon}^{\mu} = \begin{pmatrix} \varepsilon^{\mu} & 0 & 0 \\ 0 & -\frac{1}{2}\varepsilon^{\mu} & 0 \\ 0 & 0 & -\frac{1}{2}\varepsilon^{\mu} \end{pmatrix}. \tag{15}$$

This strain is considered as an eigenstrain in the mechanical problem, meaning the strain that would appear in the absence of the surrounding matrix. Either analytical models can be used. A classical model is the piezomagnetic one: $\varepsilon^{\mu} = \alpha \|\mathbf{B}^{\mathbf{ms}}\|$.

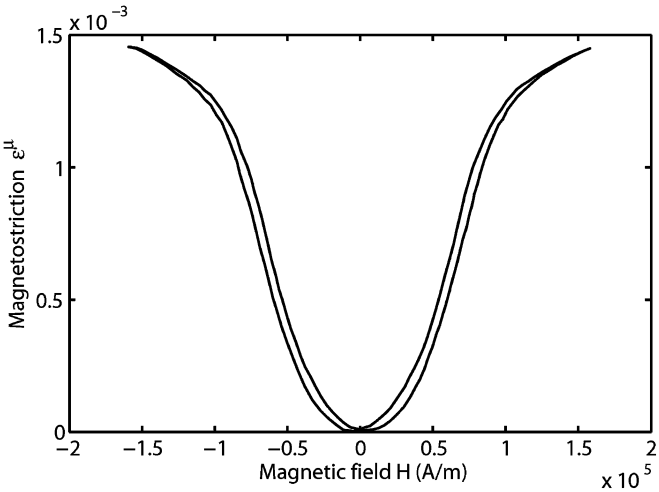

Fig. 4. Experimental magnetostriction curve for Terfenol-D [13] (no applied stress).

Some authors propose a more refined model for magnetostriction strain (see for example [12], [15], [16]), with better agreement to experimental data

$$\varepsilon^{\mu} = \alpha \mathbf{B}^{\mathbf{ms}2}. \tag{16}$$

Either experimental data are available (see Fig. 4), this solution would lead to the most accurate model.

The next step is then the calculation of the macroscopic mechanical state depending on the magnetostriction strain.

B. Mechanical Calculation

The mechanical homogenization step in this problem is slightly different from the one presented in Section II because the loading is not the macroscopic external one in that case but the local eigenstrain previously computed.

Strain is broken up into two terms: $\boldsymbol{\epsilon} = \boldsymbol{\epsilon}^e + \boldsymbol{\epsilon}^{\mu}$ where $\boldsymbol{\epsilon}$ is the total strain, $\boldsymbol{\epsilon}^e$ the purely elastic strain, and $\boldsymbol{\epsilon}^{\mu}$ the magnetostrictive strain. Macroscopically, the same kind of relation exists: $\boldsymbol{E} = \boldsymbol{E}^e + \boldsymbol{E}^{\mu}$. But these quantities are not simply linked in the general case: $\bar{\boldsymbol{E}} = \langle \boldsymbol{\epsilon} \rangle$ but $\bar{\boldsymbol{E}}^e \neq \langle \boldsymbol{\epsilon}^e \rangle$ and $\bar{\boldsymbol{E}}^{\mu} \neq \langle \boldsymbol{\epsilon}^{\mu} \rangle$. Indeed, the macroscopic magnetostriction strain can be computed [17]:

$$\boldsymbol{E}^{\mu} = \left\langle {}^t\mathbb{B}^i : \boldsymbol{\varepsilon}^{\mu i} \right\rangle \neq \langle \boldsymbol{\varepsilon}^{\mu} \rangle \tag{17}$$

with $\mathbb{B}^i$ the localization tensor[3] linking local stress $\boldsymbol{\sigma}^i$ to macroscopic stress $\boldsymbol{\Sigma}$ ($\boldsymbol{\sigma}^i = \mathbb{B}^i : \boldsymbol{\Sigma}$). This tensor is equal to

$$\mathbb{B}^i = \mathbb{C}^i : (\mathbb{C}^* + \mathbb{C}^i)^{-1} : (\mathbb{C}^* + \widetilde{\mathbb{C}}) : \widetilde{\mathbb{C}}^{-1} \tag{18}$$

with $\widetilde{\mathbb{C}}$ the effective stiffness tensor depending on the choice of the infinite medium. Since the matrix is nonmagnetostrictive

$$\boldsymbol{E}^{\mu} = f^t\mathbb{B}^{ms} : \boldsymbol{\varepsilon}^{\mu}. \tag{19}$$

[3]The index ${}^t\mathbb{X}$ denotes a transposition operation on $\mathbb{X}$.

Several estimates for the effective stiffness tensor can be chosen [7]. The most usual choices are the dilute model (also called Eshelby model), the Hashin & Shtrikman model, or the Mori-Tanaka model. In the studied case, analytical expression can be found for these estimates. We will focus in that paper on the Hashin & Shtrikman estimate where the infinite medium is chosen with the same properties as the matrix of the composite. In that case, the effective tensor is given by (20)

$$\widetilde{\mathbb{C}}^{HS} = \left(f(\mathbb{C}^* + \mathbb{C}^{ms})^{-1} + (1-f)(\mathbb{C}^* + \mathbb{C}^m)^{-1}\right)^{-1} - \mathbb{C}^*. \tag{20}$$

If the composite is not constrained on its border, the macroscopic stress free strain has an analytical expression given by (21)

$$\boldsymbol{E}^{HS} = \frac{5fG^{ms}(3k^{mat} + 4G^{mat})}{D}\boldsymbol{\varepsilon}^{\mu} \tag{21}$$

where

$$D = \left(fG^{ms} + (1-f)G^{mat}\right)(9k^{mat} + 8G^{mat}) + 6G^{ms}(k^{mat} + 2G^{mat}). \tag{22}$$

C. Application on Terfenol-D Composites

Let us first consider a Terfenol-D/glass composite. The model enables the computation of macroscopic stress free strain of the composite from an applied macroscopic magnetic field. The following application consists in the computation of the composite stress free strain for different macroscopic applied fields and different Terfenol-D volumetric fractions.

The mean magnetic field in Terfenol-D particles is computed (in the nonlinear case, the magnetization curve is required, anhysteretic curves are used for Terfenol-D). The composite magnetization curve can also be deduced (Fig. 5).

Then, the magnetostriction response of pure Terfenol-D has to be determined. Either from experimental data giving the magnetization curve (Fig. 3) and the α coefficient (2.3×10^{-3} for Terfenol-D according to [12]), either directly from experimental data on magnetostriction (Fig. 4). This latter choice has been made for the applications in that paper. Finally, the stress-free strain can be computed for different applied fields and volumetric fractions (Fig. 6).

These results can be compared to experimental ones given in [18] and used in [5]. Qualitatively, results show a good agreement with experiment but quantitatively, there is a significant difference. This difference comes from the magnetostriction response of pure Terfenol-D: Zhou *et al.* deduced the pure Terfenol-D response from experimental data on a 60% Terfenol-D composite, whereas we only used properties from the different constituents. Moreover, the level of applied magnetic field in this publication appears to be very high compared to usual experimental levels. Anyway, if we use the Terfenol-D magnetostriction curve proposed by Zhou *et al.*, the agreement between experimental and modeling results becomes very good (Fig. 7).

The model can also be applied to Terfenol-D/Epoxy composites. In most cases, these composites are made of $\langle 112 \rangle$ oriented particles of Terfenol-D. The behavior of such particles

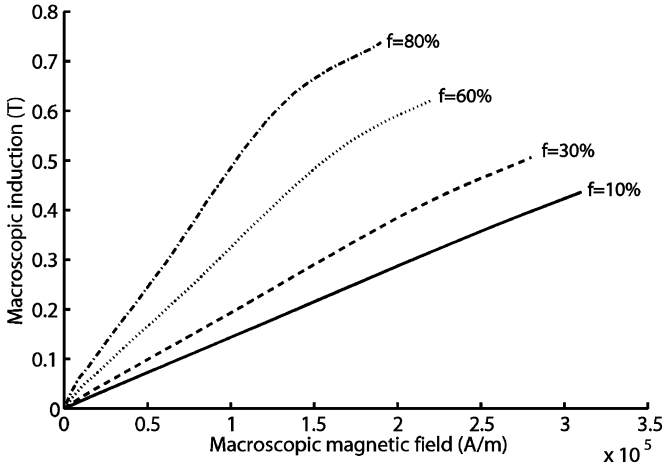

Fig. 5. Anhysteretic magnetization curve of the Terfenol-D/glass composite for different volumetric fractions. Modeling results.

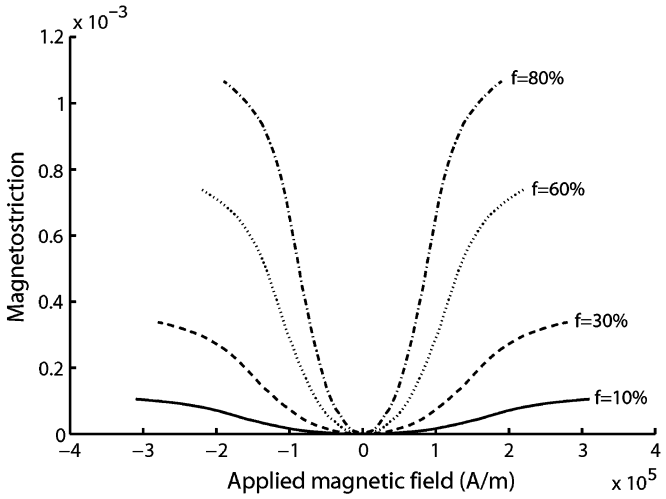

Fig. 6. Macroscopic stress free strain of the Terfenol-D/glass composite for different volumetric fractions. Modeling results.

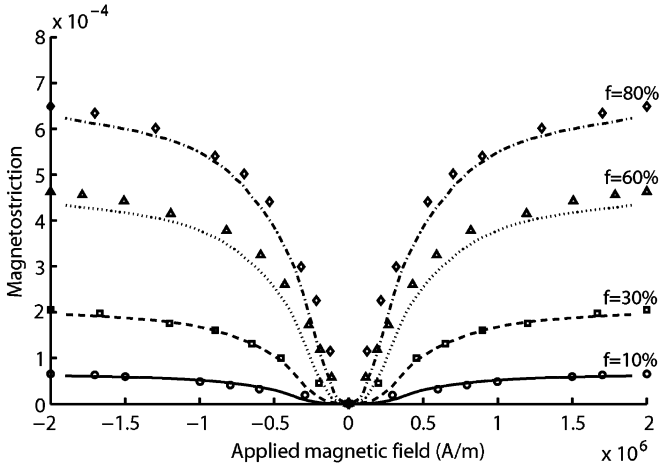

Fig. 7. Experimental (symbols) and modeling (lines) results for the Terfenol-D/glass composite using the magnetostriction response proposed in [5] for different volumetric fractions.

is close to the behavior of $\langle 112 \rangle$ single crystals. Magnetization and magnetostriction measurements on Terfenol-D single crystals have been reported in [19]. These results have been used for the modeling. The effect of the volumetric fraction of Terfenol-D is plotted in Fig. 8.

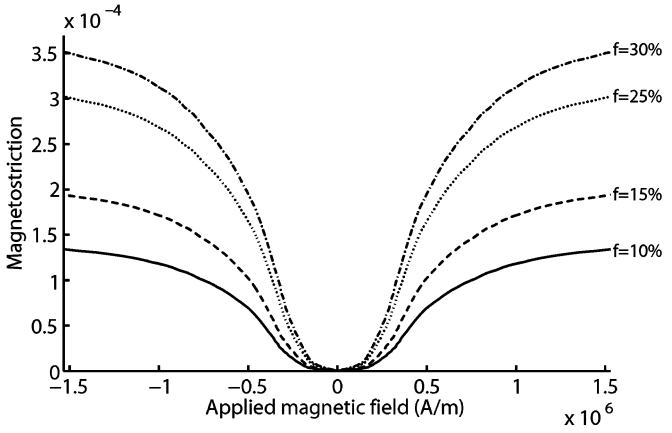

Fig. 8. Macroscopic stress free strain of the Terfenol-D/Epoxy composite for different volumetric fractions. Modeling results.

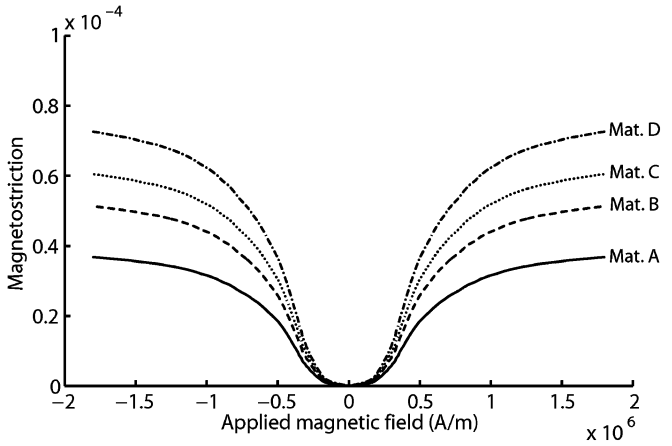

Fig. 9. Macroscopic stress free strain of a 10% Terfenol-D composite for different matrices. Modeling results.

These results are in good qualitative agreement with the experimental results reported by Anjanappa and Wu [20], revealing an increase of the magnetostriction strain with the volumetric fraction. But they strongly differ from the experimental results from Duenas and Carman [21]. They report a nonmonotonous effect of the volumetric fraction on the magnetostriction amplitude: the maximum amplitude is higher for a 20% composite than for a 30% one. The proposed model cannot capture such an effect. However, the reason for such an effect would have to be clearly identified to be modeled. The effect of residual stresses arising from the manufacturing process is an hypothesis in the particular case reported in this reference.

The effect of the matrix properties can also be investigated. The estimated response of a 10% Terfenol-D composite with different materials for the matrix is presented in Fig. 9. The material properties used are given in Table I. The results fit well the experimental results reported in [18] and plotted in Fig. 10.

IV. Optimal Design of Magnetostrictive Composite

Until that point, composites have been considered unconstrained. This hypothesis is an extreme case. In most practical

TABLE I
MATERIAL YOUNG MODULUS [18]

Material	Young modulus
Terfenol-D	30 GPa
Cu + glass (matrix A)	104 GPa
Al + glass (matrix B)	65 GPa
Glass (matrix C)	50 GPa
$CeFe_2$ + glass (matrix D)	36 GPa

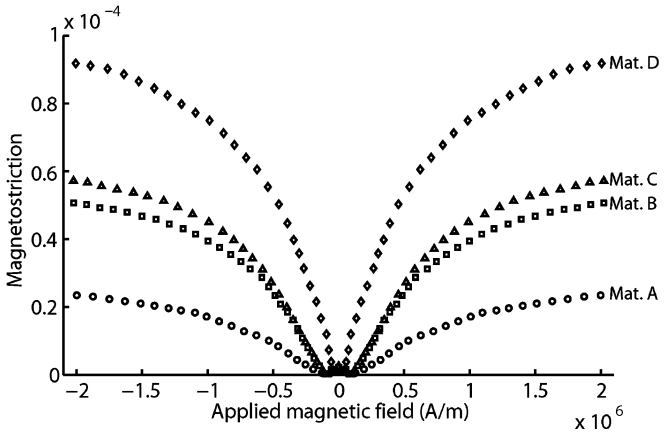

Fig. 10. Macroscopic stress free strain of a 10% Terfenol-D composite for different matrices. Experimental results [18].

cases, the stress is not zero. One other extreme case is the one where the composite cannot deform. Then, the blocked stress (compression) can be deduced. The actual behavior stands between these two cases (the unconstrained strain and the blocked stress). In the present model, the behavior can be represented with a straight line (going through the previous two points) since we consider a linear mechanical behavior and a stress-independent magnetic state. Then, an analytical model can be given for the characteristic curve of the composite. The operating point can be deduced from the crossing between this characteristic curve and the loading application one. Fig. 11 shows an example of application for which the loading curve is linear. The intersection point is the actual operating point.

The need for optimal design for magnetostrictive composites can be pointed out with the sensitivity of some parameters such as magnetostrictive phase volumetric fraction or matrix stiffness. It can be noticed in (21) that composite strain or stress are proportional to the magnetostriction strain of Terfenol-D particles. As a consequence, in the following figures, a modified applied field would not change the shape of the curves but only the axes values. For a given set of parameters (volumetric fraction, matrix shear and bulk modulus—or Young modulus and Poisson ratio—and applied field), the behavior of the composite is shown in Fig. 11.

For the same volumetric fraction, let us consider different Young modulus values for the matrix. The corresponding characteristic curves are presented in Fig. 12, defining the envelope of the reachable behaviors. This figure points out the fact that any point (depending on the application) under the limit curve can be reached with the good choice of the matrix Young modulus for a given volumetric fraction. This limit curve gives the optimum operating points for the composite actuator. Using that

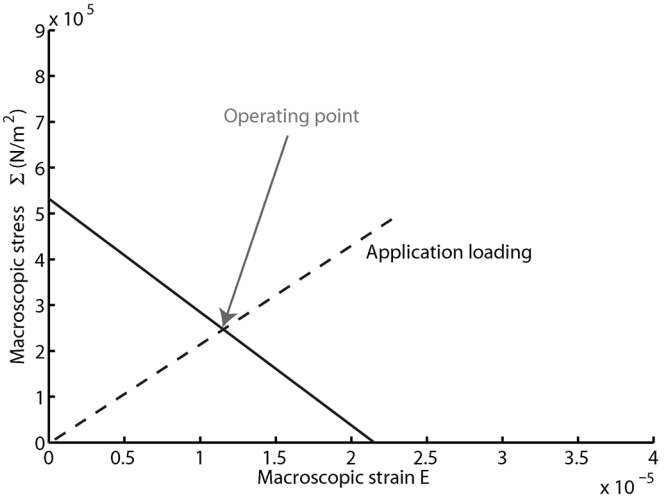

Fig. 11. Macroscopic strain versus macroscopic stress (compression) for one set of parameters.

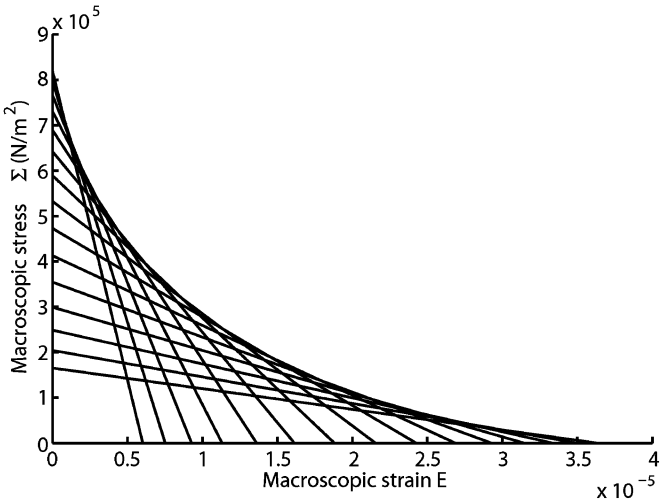

Fig. 12. Macroscopic strain versus macroscopic stress for different matrix Young modulus (from 5 to 200 GPa), $f = 10\%$ and $\overline{\mathbf{H}} = 100$ kA/m.

curve, the best use of the composite can be deduced. The volumetric fraction effect can also be shown in Fig. 13.

Magnetostrictive phase volumetric fraction can be optimized, for example if the maximum attainable magnetic field is given. In order to design cheaper composites, the minimum magnetostrictive phase volumetric fraction can be found with the help of Fig. 13. The effect of the matrix Poisson ratio can also be shown but it is a lot less sensitive than volumetric fraction and Young modulus.

These results point out the effect of the parameters related to the composition and microstructure on the overall composite behavior. With the knowledge of the application and the applied load range (magnetic and mechanical) on the actuator, the choice of the matrix properties and volumetric fractions can be considerably optimized.

V. CONCLUSION

The behavior of magnetostrictive composites has been modeled through homogenization tools from both a magnetic and a mechanical point of view. The use of this model can lead to the

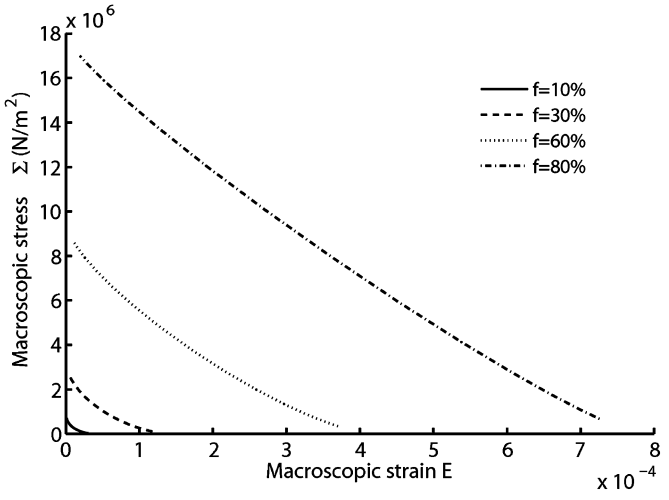

Fig. 13. Optimum operating curves for the same previous Terfenol-D volumetric fractions, $\overline{\mathbf{H}} = 100$ kA/m.

optimization of composites behavior. Sensitivity to volumetric fractions of the magnetostrictive phase or to Young modulus of the matrix have been presented and really affect the composite properties. Results have been presented for biphasic composites with one magnetostrictive phase (for which experimental data are available) but this model can apply to the study of composites with several magnetostrictive phases without any change in the presented method.

Moreover, this model takes into account the nonlinear magnetic behavior of magnetostrictive phases. Indeed the evaluation of the local magnetic field in magnetostrictive particles highly depends on it and is a key point. Other models often consider linear magnetic behavior, implying an error on local magnetic field evaluation and as a consequence, an error on eigenstrain evaluation.

One limitation of this model is the fact that the magnetostriction of magnetostrictive particles is just magnetic field dependent. In practical cases, it is also known to be stress dependent. Thus, the model can be used to get optimal composition of a composite, but is not able to catch second-order effects such as the effect of residual stress due to thermal treatments. The introduction of stress in the magnetostrictive constitutive law is currently a work in progress. This improvement could be based on recent works on magneto-mechanical behavior modeling including the multiaxiality of stress [22].

This model is nevertheless a fast and useful tool for the first step design of composition and microstructure of magnetostrictive composites.

REFERENCES

[1] G. Engdahl, *Handbook of Giant Magnetostrictive Materials*. New York: Academic, 2000.

[2] J. F. Herbst, T. W. Capehart, and F. E. Pinkerton, "Estimating the effective magnetostriction of a composite: A simple model," *Appl. Phys. Lett.*, vol. 70, no. 22, pp. 3041–3043, Jun. 2, 1997.

[3] C.-W. Nan, "Effective magnetostriction of magnetostrictive composites," *Appl. Phys. Lett.*, vol. 72, no. 22, pp. 2897–2899, Jun. 1, 1998.

[4] C.-W. Nan and G. J. Weng, "Influence of microstructural features on the effective magnetostriction of composite materials," *Phys. Rev. B*, vol. 60, no. 9, pp. 6723–6730, 1999.

[5] Y. Zhou and F. G. Shin, "Modeling of magnetostriction in particulate composite materials," *IEEE Trans. Magn.*, vol. 41, no. 6, pp. 2071–2076, Jun. 2005.

[6] X. Feng, D.-N. Fang, and K.-C. Hwang, "Closed-form solutions for piezomagnetic inhomogeneities embedded in a non-piezomagnetic matrix," *Eur. J. Mech. A—Solids*, vol. 23, pp. 1007–1019, 2004.

[7] M. Bornert, T. Bretheau, and P. Gilormini, *Homogenization in Mechanics of Materials*. London, U.K.: Iste, 2007.

[8] A. Sihvola, *Electromagnetic Mixing Formulas and Applications*. London, U.K.: IEE Electromagnetic Waves Series 47, 1999.

[9] L. Daniel and R. Corcolle, "A note on the effective magnetic permeability of polycrystals," *IEEE Trans. Magn.*, vol. 43, no. 7, pp. 3153–3158, Jul. 2007.

[10] J. A. Stratton, *Electromagnetic Theory*. New York: McGraw-Hill, 1941.

[11] B. D. Cullity, *Introduction to Magnetic Materials*. Reading, MA: Addison-Wesley, 1972.

[12] K. Azoum, M. Besbes, F. Bouillault, and T. Ueno, "Modeling of magnetostrictive phenomena. application in magnetic force control," *Eur. Phys. J. Appl. Phys.*, vol. 36, no. 1, pp. 43–47, 2006.

[13] A. E. Clark, M. L. Spano, and H. T. Savage, "Effect of stress on the magnetostriction and magnetization of rare earth- $\mathrm{Re}_{1,95}$ alloys," *IEEE Trans. Magn.*, vol. MAG-19, no. 5, pp. 1964–1966, Sep. 1983.

[14] E. du Trémolet de Lacheisserie, *Magnetostriction—Theory and Applications of Magnetoelasticity*. Boca Raton, FL: CRC, 1993.

[15] D. C. Jiles, "Theory of the magnetomechanical effect," *J. Phys. D: Appl. Phys.*, vol. 28, no. 8, pp. 1537–1546, 1995.

[16] K. Delaere, W. Heylen, R. Belmans, and K. Hameyer, "Strong magnetomechanical FE coupling using local magnetostriction forces," *Eur. Phys. J. Appl. Phys.*, vol. 13, no. 2, pp. 115–119, 2001.

[17] D. François, A. Pineau, and A. Zaoui, *Mechanical Behaviour of Materials: Elasticity and Plasticity*. Norwell, MA: Kluwer, 1998.

[18] Y. Chen, J. E. Snyder, C. R. Schwichtenberg, K. W. Dennis, D. K. Falzgraf, R. W. McCallum, and D. C. Jiles, "Effect of the elastic modulus of the matrix on magnetostrictive strain in composites," *Appl. Phys. Lett.*, vol. 74, no. 8, pp. 1159–1161, Feb. 22, 1999.

[19] B. W. Wang, S. C. Busbridge, Y. X. Li, G. H. Wu, and A. R. Piercy, "Magnetostriction and magnetization process of $\mathrm{Tb}_{0.27}\mathrm{Dy}_{0,73}\mathrm{Fe}_2$ single crystal," *J. Magn. Magn. Mater.*, vol. 218, pp. 198–202, 2000.

[20] M. Anjanappa and Y. Wu, "Magnetostrictive particulate actuators: Configuration, modeling and characterization," *Smart Mater. Struct.*, vol. 6, pp. 393–402, 1997.

[21] T. A. Duenas and G. P. Carman, "Large magnetostrictive response of terfenol-D resin composites (invited)," *J. Appl. Phys.*, vol. 87, no. 9, pp. 4696–4701, May 1, 2000.

[22] L. Daniel, N. Buiron, O. Hubert, and R. Billardon, "Reversible magneto-elastic behavior: A multiscale approach," *J. Mech. Phys. Solids.* 2007 [Online]. Available: http://dx.doi.org/10.1016/j.jmps.2007.06.003

Manuscript received June 6, 2007; revised October 12, 2007. Corresponding author: L. Daniel (e-mail: laurent.daniel@lgep.supelec.fr).