



Energetical and multiscale approaches for the definition of an equivalent stress for magneto-elastic couplings

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ABSTRACT

A main limitation of most models describing the effect of stress on the magnetic behavior is that they are restricted to uniaxial – tensile or compressive – stress. Nevertheless, stress is multiaxial in most of industrial applications. An idea to overcome the strong limitation of models is to define a fictive uniaxial stress, the equivalent stress, that would change the magnetic behavior in a similar manner than a multiaxial stress. A first definition of equivalent stress, called the deviatoric equivalent stress, is proposed. It is based on an equivalence in magneto-elastic energy. This formulation is first derived for isotropic materials under specific assumptions. An extension to orthotropic media under disoriented magneto-mechanical loading is made. A new equivalent stress expression, called generalized equivalent stress, is then proposed. It is based on an equivalence in magnetization. Inverse identification of equivalent stress is made possible thanks to a strong simplification of the description of the material seen as an assembly of elementary magnetic domains. It is shown that this second proposal is a generalization of the deviatoric expression. Equivalent stress proposals are compared to former proposals and validated using experimental results carried out on an iron–cobalt sheet submitted to biaxial mechanical loading. These results are compared to the predictions obtained thanks to the equivalent stress formulations. The generalized equivalent stress is shown to be a tool able to foresee the magnetic behavior of a large panel of materials submitted to multiaxial stress.

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1. Introduction

In most practical electromagnetic applications, magnetic materials are submitted to multiaxial stress inherited from forming process or appearing in use. As an example, inertial stresses in high rotating speed systems for aeronautic equipments or new technologies of flywheel, stresses due to binding process (encapsulation) of electrical machines and actuators, residual stress associated to plastic straining (forming) or cutting process can be mentioned. Several examples that can be associated to some generic types of stress tensor are detailed in this paper. On the other hand, since the works of Mateucci [1] and Villari [2], mechanical stress is known to change significantly the magnetic behavior of materials (see for instance [3]) as well as their magnetostrictive behavior [4]. The design of electromagnetic systems consequently requires coupled and multiaxial models. However, the few available and practically implemented models describing the effect of stress on the magnetic behavior are restricted to uniaxial (tensile or compressive) stress.

Jiles–Atherton type models [5–8] and Preisach type models [9–12] are the most popular but other approaches have also been proposed [13–15].

One way is to use energy-based models written at an appropriate scale. Indeed the development of fully multiaxial magneto-elastic models is a promising issue [16–22], but still leads to dissuasive computation times for engineering design applications. The second part of this paper briefly recalls such a multiscale magneto-elastic model for the prediction of the effect of multiaxial stress on magnetic and magnetostrictive behavior.

A second way is to implement multiaxial stress directly in a macroscopic model (see for instance [23]). Even sometimes effective, this solution is not fully satisfactory since the physics of coupling is not suitably described. Another approach combining uniaxial models and multiaxial stress is to define and calculate a “fictive” uniaxial stress, the equivalent stress that would change the magnetic behavior in a similar manner than the multiaxial one. The second step is to implement this scalar value into a uniaxial macroscopic magneto-mechanical model. Some authors proposed such an approach in the past years [24–27]. It is shown that in many cases these proposals are not fully satisfactory. Recently a definition of an equivalent stress based on an equivalence in magneto-elastic energy has been proposed [28,29]. Assuming that

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the same magneto-elastic energy corresponds to the same magnetic behavior, the equivalent stress is defined as the uniaxial stress, applied along the magnetic field direction that leads to the same macroscopic magneto-elastic energy as the multiaxial one. This formulation is named as the deviatoric equivalent stress because it is expressed in terms of the deviatoric stress tensor. Its implementation is very simple; it is nevertheless involving some strong hypotheses on the behavior of the material and on the magneto-mechanical loading. Extension to orthotropic media and to a disoriented loading is proposed in this paper in order to solve some of the limitations of this approach. In the next section, a generalized equivalent stress is proposed, based on an equivalence in magnetization. Since magnetostatic and magnetoelastic terms cannot be easily dissociated in the expression of the free energy, the definition of the equivalent stress is made possible thanks to a strong simplification in the description of the material. Bulk magnetic material is seen as an assembly of elementary magnetic domains. The validation of these proposals requires to carry out magnetic measurements on materials submitted to multiaxial stress thanks to a multiaxial experimental set-up. The proposed validation is based on biaxial experiments.

Biaxial testing in mechanics is usually associated to traction-torsion, internal pressure, and/or traction-traction (along two orthogonal axes) experiments. Considering that sheet format is the most popular format for magnetic materials constitutive of electrical machines [3], only the latter experiment is suitable to study the magnetic behavior under biaxial mechanical conditions. Previous experiments from different authors are available in the literature (see for instance [24–26,30] and more recently [31–33]). But all of them present some mismatches especially concerning the homogeneity of the stress in the measurement area (e.g. Langman's eight points bending system [30]) or the inaccurate evaluation of magnetic quantities [33]. A previous paper detailed a new experimental procedure and corresponding results [34,35]. The procedure, partially recalled in this paper, allowed to evaluate the influence of a biaxial stress state on the magnetic behavior under adequate magnetic and mechanical conditions.

The new experimental results have been used in order to test the validity of previous and new formulations of equivalent stress. A short review of previous experimental results is finally made. These results are compared to predictions obtained thanks to the new formulation of equivalent stress. The generalized equivalent stress is shown to be a tool able to foresee the magnetic behavior of a large panel of materials submitted to multiaxial stress.

The paper is divided into five sections. In the first part, some typical multiaxial stress states that can be encountered in electromagnetic devices are presented. Magneto-elastic modeling approaches able to account for the effect of multiaxial stress on the

magnetic behavior are presented in the second part. In the third and fourth parts, two distinct proposals for the definition of an equivalent stress for the magnetic behavior are proposed: the deviatoric and the generalized equivalent stresses. A validation of these approaches is finally proposed by comparison to biaxial magneto-mechanical measurements.

2. Multiaxial stress in magnetic materials and structures

Forces, torques, stresses acting on the material are associated to any operating electromagnetic system. Multiaxial stresses are generally created due to the nature of loading, to the complex geometry of devices or to the manufacturing process. Several examples of typical loadings and associated stress tensors are given in this section. Elastic mechanical behavior is assumed.

2.1. Multiaxial stress “in use”: centrifugal forces and torque

The first example concerns the effect of centrifugal forces. Indeed modern technologies of electrical machines especially for aeronautical applications involve higher and higher rotating speed and torque. Centrifugal forces are also critical in new technologies of high speed flywheels (Fig. 1) [36].

Let us consider a ferromagnetic full cylinder (external diameter ϕ_e —Fig. 2) submitted to a constant angular speed $\vec{\omega} = \omega \cdot \vec{e}_z$. The volumetric forces are radial and related to the radius r by $\vec{f}_v = \rho r \omega^2 \cdot \vec{e}_r$ (ρ is the mass density of the considered material). The corresponding stress tensor σ_0 is diagonal (1). The definition of each term depends on the assumption on the stress and strain tensors. Plane stress approximation or plane strain approximation leads to two classical results in continuum

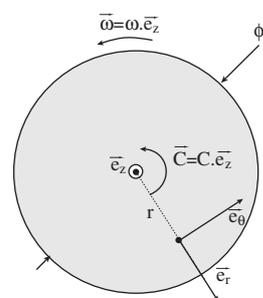


Fig. 2. Schematic view of a ferromagnetic full cylinder submitted to angular speed $\vec{\omega}$ and/or torque \vec{C} .

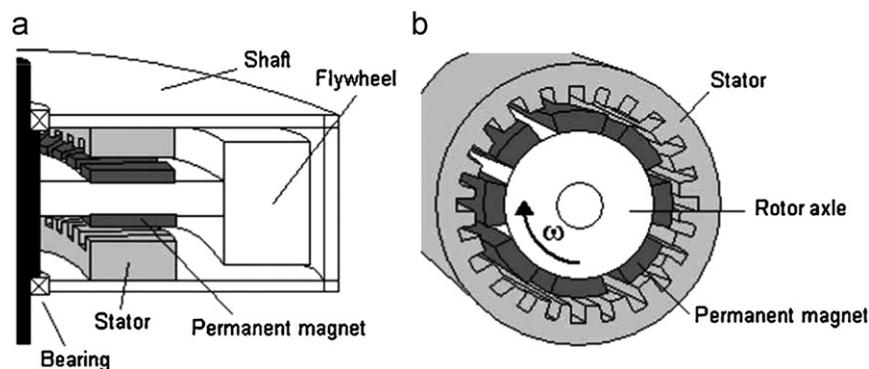


Fig. 1. Examples of flywheel systems: (a) axial-flux and, (b) radial-flux permanent magnet machines [36].

mechanics:

$$\sigma_0 = \begin{pmatrix} \sigma_{rr} & 0 & 0 \\ 0 & \sigma_{\theta\theta} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}_{(r,\theta,z)} \quad (1)$$

- In the case of plane stress conditions (free deformation through the thickness), the stress tensor is biaxial and its components are given by

$$\begin{cases} \sigma_{rr} = -K' \cdot (3-2\nu-4\nu^2) \left(r^2 - \left(\frac{\phi_e}{2} \right)^2 \right) \\ \sigma_{\theta\theta} = -K' \cdot \left((1+2\nu-4\nu^2)r^2 - (3-2\nu-4\nu^2) \left(\frac{\phi_e}{2} \right)^2 \right) \\ \sigma_{zz} = 0 \end{cases}$$

- In the case of plane strain conditions:

$$\begin{cases} \sigma_{rr} = -K' \cdot (3-2\nu) \left(r^2 - \left(\frac{\phi_e}{2} \right)^2 \right) \\ \sigma_{\theta\theta} = -K' \cdot \left((1+2\nu)r^2 - (3-2\nu) \left(\frac{\phi_e}{2} \right)^2 \right) \\ \sigma_{zz} = -2\nu K' \left(2r^2 - (3-2\nu) \left(\frac{\phi_e}{2} \right)^2 \right) \end{cases}$$

with $K' = \rho\omega^2 / (8(1-\nu))$, ν denoting Poisson's ratio of the material. The stress is multiaxial and increases with the square of the angular speed.

The second example deals with the stress associated to the torque acting on the rotor of the rotating machine. The torque occurs during a transient period of the system, when rotating speed is increasing or decreasing. Stress due to pure torque is usually added to the stress due to centrifugal forces.

Let us consider the previous ferromagnetic full cylinder (Fig. 2) submitted to a constant torque $\vec{C} = C \cdot \vec{e}_z$. The associated stress tensor σ_1 is null except the shear term $\sigma_{\theta z}$:

$$\sigma_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \sigma_{\theta z} \\ 0 & \sigma_{\theta z} & 0 \end{pmatrix}_{(r,\theta,z)} \quad (2)$$

with

$$\sigma_{\theta z} = \frac{8rC}{\pi\phi_e^2}$$

r denoting the radial position (Fig. 2). Such a shear stress is associated to a biaxial principal stress tensor, with opposite eigen stresses $\sigma^E = \pm \sigma_{\theta z}$.

2.2. Initial multiaxial stress due to forming process: binding and coiling processes

Multiaxial stress can occur during the forming process of the magnetic material, during the assembly of the electrical machine, or during the machining of a part of the machine (usually the magnetic sheets).

The binding process is usually employed to stack together the magnetic sheets of the stator of an electrical machine. It also ensures a protection of the magnetic circuit. This binding is usually realized on an axisymmetric stator: the ring is heated to dilate and placed around the stator. The thermal contraction ensures the mechanical anchoring (see Fig. 3).

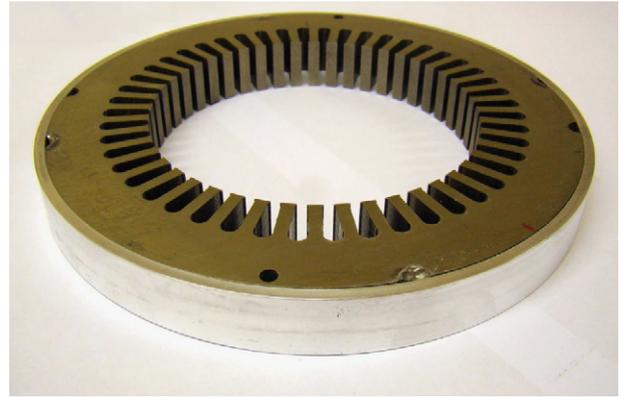


Fig. 3. Example of binding of a stator using an aluminum ring.

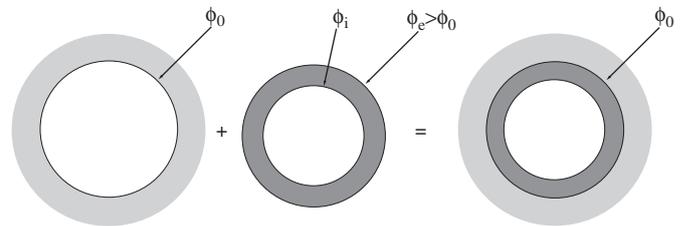


Fig. 4. Theoretical binding of a cylinder inside a rigid yoke.

The stress due to binding can be calculated under plane strain assumption, and using simplified geometric conditions. Let us consider a rigid cylindrical yoke of internal diameter ϕ_0 , binding an iron cylinder of external diameter $\phi_e = \phi_0 + 2\delta$ and internal diameter ϕ_i (Fig. 4). Binding strength depends on these three geometric parameters and on material constants (the so-called Lamé coefficients μ and λ in the case of isotropic elasticity). After calculation we get the stress tensor σ_2 as a function of the radius r (Eq. (3)). The stress state is triaxial, non-homogeneous and linearly depends on δ parameter:

$$\sigma_2 = \begin{pmatrix} \sigma_{rr} & 0 & 0 \\ 0 & \sigma_{\theta\theta} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}_{(r,\theta,z)} \quad (3)$$

with

$$\begin{cases} \sigma_{rr} = -K\delta \left(4 - \left(\frac{\phi_0}{r} \right)^2 \right) \\ \sigma_{\theta\theta} = -K\delta \left(4 + \left(\frac{\phi_0}{r} \right)^2 \right) \\ \sigma_{zz} = -K\delta \left(\frac{4\lambda}{\mu + 2\lambda} \right) \end{cases}$$

and

$$K = \frac{\mu(\mu + 2\lambda) \cdot \phi_e}{\mu(\phi_e^2 + \phi_i^2) + 2\lambda\phi_i^2}$$

Multiaxial stress can also appear in a magnetic material after plastic straining. Associated stresses are commonly called “internal stresses” if the scale of fluctuation of stress is lower than the grain size and “residual stresses” if the scale of fluctuation of stress is of the order of the specimen size. Effect of plasticity on magnetic materials can be interpreted as an effect of these internal or residual stresses [37,38]. Plastic strains occur for example at the cutting edge of sheets [39,40], or after bending during the forming process. We can consider the case of the

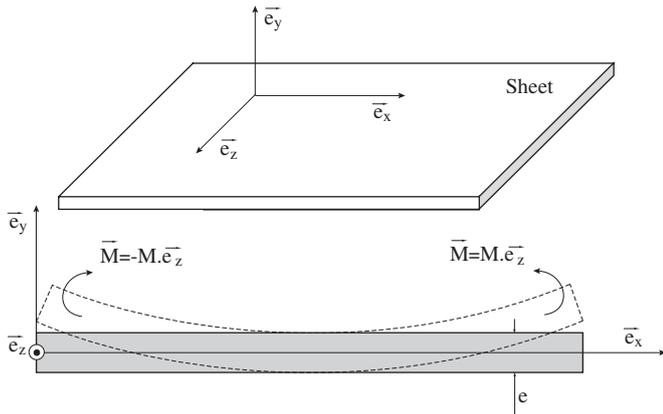


Fig. 5. Schematic view of a plane sheet submitted to a torque $\vec{M} = \pm M \cdot \vec{e}_z$.



Fig. 6. Residual curvature of an iron–cobalt sheet after severe coiling curvature denotes internal stresses.

coiling of a sheet around a cylinder: an irreversible deformation may occur if the diameter of the cylinder is too small. The mechanical state associated to the coiling is complex. A usual simplification plotted in Fig. 5 is to consider a pure bending of the sheet (thickness e and plane (\vec{e}_z, \vec{e}_x)) due to a torque $\vec{M} = \pm M \cdot \vec{e}_z$ associated to a dry friction condition on the lower surface (relative slip speed considered as zero). We can evaluate a stress tensor σ_3 through the thickness of the sheet (depth y) as follows:

$$\sigma_3 = \begin{pmatrix} K''(y) \cdot (2\mu + \lambda) & 0 & 0 \\ 0 & K''(y) \cdot \lambda & 0 \\ 0 & 0 & K''(y) \cdot \lambda \end{pmatrix}_{(x,y,z)} \quad (4)$$

with

$$K''(y) = \frac{M}{EI} \left(y + \frac{e}{2} \right)$$

E is the Young's modulus and I the moment of inertia of the section (\vec{e}_y, \vec{e}_z) around \vec{e}_z axis. When the yield stress is reached on the upper surface, stress tensor brings multiaxial plasticity, leading to residual stresses after unloading. Fig. 6 shows an iron–cobalt sheet deformed by bending after a coiling.

The previous simple examples show the need of magneto-elastic models considering multiaxial macroscopic stresses in order to predict their effects and optimize the design of electromagnetic devices. In industrial magnetic systems, the distribution of stress can be more complex and is often estimated using finite element techniques (see for instance [41,42]).

3. Multiaxial magneto-elastic models

The prediction of the influence of multiaxial stress on magnetic behavior supposes the introduction of the complete mechanical loading into a magnetoelastic modeling. As already mentioned, the few practically implemented models describing the effect of stress on magnetic behavior are restricted to uniaxial mechanical loadings (tension or compression) [5–15]. A first approach to build multiaxial magneto-elastic models consists in the definition of magneto-elastic constitutive laws including the

multiaxiality of stress at the local scale. A multiscale model following this requirement is briefly presented in the first part. A second approach consists in the definition of a uniaxial equivalent stress, defined from the multiaxial loading and implemented into a uniaxial constitutive law. Such an approach is presented in the second part.

3.1. Multiaxial magneto-elastic constitutive laws: the multiscale model

At lower scales, micromagnetics offers a deep insight on the dynamics of magnetic structures—magnetic domains and domain walls. Micromagnetic approaches [43] are based on the local resolution of Landau–Lifshitz–Gilbert equation of motion often in combination with finite element methods [44]. In order to account for magneto-elastic coupling, the elastic energy can be introduced with respect to the balance equations and the boundary conditions [45–48]. This point remains a complex issue particularly in the context of multiaxial stress. Despite an accurate prediction of local magnetic microstructures, the small volumes considered and the high computation time associated to these calculations remain a significant drawback.

Following the early works of Néel [49], the development of multiscale magneto-elastic models is a promising issue [16–22]. These models are inspired from the equations of micromagnetics with the use of an energetic functional to be minimized but take benefit from the results of micromagnetic calculations in order to define simplifying assumptions. The computation times are then significantly improved. The multiaxiality of stress can usually be naturally accounted for in such approaches. Such a multiscale model is briefly presented hereafter. It is a three scale model – domain, grain and macroscopic scales – for the prediction of the reversible magneto-elastic behavior of heterogeneous materials [21,22].

A ferromagnetic medium can be seen as an aggregate of single crystals assembled following an orientation distribution function. Each single crystal can itself be seen as an aggregate of magnetic domains following another distribution function. The single crystal can be divided into domain families, each family α corresponding to a given orientation for the magnetization. The modeling scheme is based on the calculation of the volumetric fraction f_α of each domain family α . The volumetric fraction corresponding to the domain family α depends on the internal energy W_α classically defined (Eq. (5)) as the sum of the magneto-crystalline (Eq. (6)), Zeemann (Eq. (7)) and elastic (Eq. (8)) energies:

$$W_\alpha = W_\alpha^K + W_\alpha^H + W_\alpha^\sigma \quad (5)$$

$$W_\alpha^K = K_1(\gamma_1^2\gamma_2^2 + \gamma_2^2\gamma_3^2 + \gamma_3^2\gamma_1^2) + K_2(\gamma_1^2\gamma_2^2\gamma_3^2) \quad (6)$$

$$W_\alpha^H = -\mu_0 \vec{H}_\alpha \cdot \vec{M}_\alpha \quad (7)$$

$$W_\alpha^\sigma = \frac{1}{2} \sigma_\alpha : \mathbb{C}_\alpha^{-1} : \sigma_\alpha \quad (8)$$

K_1 and K_2 are the magnetocrystalline constants, μ_0 denotes the vacuum permeability. $\vec{M}_\alpha = M_s \vec{\gamma}_\alpha = M_s \gamma_i \cdot \vec{e}_i$ is the magnetization of the domain family α with M_s the saturation magnetization of the material and γ_i the direction cosines of the magnetization in the crystallographic coordinate system. \vec{H}_α and σ_α denote the magnetic field and the stress tensor at domain scale. \mathbb{C}_α is the local stiffness tensor. If no specific information is known about magnetic domain topology, the assumption of a uniform magnetic field within a grain (single crystal) can be employed. The assumption of a uniform strain within a grain leads to further simplification of the elastic energy (Eq. (9)) introducing the magnetostriction strain tensor ε_α^μ in a domain (Eq. (10)), the

average stress tensor σ_g over the grain and a constant W_0^σ [21]. This latter constant, uniform over a grain does not participate in the energetic balance and is usually removed [4,51] leading to the classical expression for the so-called magneto-elastic energy (Eq. (11)):

$$W_\alpha^\sigma = -\sigma_g : \varepsilon_\alpha^\mu + W_0^\sigma \tag{9}$$

$$\varepsilon_\alpha^\mu = \frac{3}{2} \begin{pmatrix} \lambda_{100}(\gamma_1^2 - \frac{1}{3}) & \lambda_{111}\gamma_1\gamma_2 & \lambda_{111}\gamma_1\gamma_3 \\ \lambda_{111}\gamma_1\gamma_2 & \lambda_{100}(\gamma_2^2 - \frac{1}{3}) & \lambda_{111}\gamma_2\gamma_3 \\ \lambda_{111}\gamma_1\gamma_3 & \lambda_{111}\gamma_2\gamma_3 & \lambda_{100}(\gamma_3^2 - \frac{1}{3}) \end{pmatrix} \tag{10}$$

$$W_\alpha^\sigma = -\sigma_g : \varepsilon_\alpha^\mu \tag{11}$$

λ_{100} and λ_{111} are the magnetostrictive coefficients of the single crystal. Following Chikazumi [51] and Buiron [18] the volumetric fraction f_α is calculated using a Boltzmann type function (Eq. (12)). The parameter S is defined by Eq. (13) and calculated using a spatial discretization of the possible directions for the magnetization [22]. The parameter A_s has been shown [21] to be proportional to the initial susceptibility χ_0 of the material (Eq. (14)):

$$f_\alpha = \frac{1}{S} \exp(-A_s \cdot W_\alpha) \tag{12}$$

$$S = \int_\alpha \exp(-A_s \cdot W_\alpha) d\alpha \approx \sum_\alpha \exp(-A_s \cdot W_\alpha) \tag{13}$$

$$A_s = \frac{3\chi_0}{\mu_0 M_s^2} \tag{14}$$

Magnetization \vec{M}_g and magnetostriction ε_g^μ at the grain scale are finally defined as the average values of magnetization and magnetostriction over a grain (Eqs. (15) and (16)):

$$\vec{M}_g = \langle \vec{M}_\alpha \rangle = \int_\alpha f_\alpha \vec{M}_\alpha d\alpha \approx \sum_\alpha f_\alpha \vec{M}_\alpha \tag{15}$$

$$\varepsilon_g^\mu = \langle \varepsilon_\alpha^\mu \rangle = \int_\alpha f_\alpha \varepsilon_\alpha^\mu d\alpha \approx \sum_\alpha f_\alpha \varepsilon_\alpha^\mu \tag{16}$$

This calculation has to be made for each grain of a polycrystalline aggregate. The knowledge of magneto-elastic loading at the grain scale (\vec{H}_g, σ_g) is required to process this calculation. This local loading can be obtained from the knowledge of the macroscopic loading and some assumptions on the microstructure using an appropriate micro-macro scheme. The model presented in [21,22] is based on a self-consistent scheme derived from Hill's formulation [53]. Once the response at the grain scale ($\vec{M}_g, \varepsilon_g^\mu$) is calculated, the macroscopic response (\vec{M}, ε^μ) of the material is simply obtained from averaging operations (Eqs. (17) and (18)):

$$\vec{M} = \langle \vec{M}_g \rangle \tag{17}$$

$$\varepsilon^\mu = \langle \varepsilon_g^\mu \rangle \tag{18}$$

Such an approach is fully multiaxial since the macroscopic mechanical loading σ is multiaxial. However, the corresponding computation time can be dissuasive for engineering design applications, particularly if a structural analysis is foreseen. However, in the particular case of a macroscopically isotropic material a simplified procedure can be defined [21]. It consists first in neglecting the fluctuations of magnetic field and stress over the volume (uniform magnetic field and stress assumptions). The heterogeneity of the elastic properties is also neglected. The polycrystal is then seen as an aggregate of randomly distributed magnetic domains, each macroscopic direction being considered as a possible easy direction [54]. The definition of the magnetization (Eq. (19)) and magnetostriction strain (Eq. (20)) of the

polycrystal then follows the procedure used for single crystals:

$$\vec{M} = \int_\alpha f_\alpha \vec{M}_\alpha d\alpha \tag{19}$$

$$\varepsilon^\mu = \int_\alpha f_\alpha \varepsilon_\alpha^\mu d\alpha \tag{20}$$

Such a simplified description can provide analytical results in certain particular configurations and will be helpful for the definition of the generalized equivalent stress.

3.2. Equivalent stress methods

Another way to account for the multiaxiality of stress in magneto-elastic modeling is to define an equivalent stress. An equivalent stress for magnetic behavior is a fictive uniaxial stress that would change the magnetic behavior in a similar manner than the real multiaxial one. This equivalent stress can be implemented into a macroscopic uniaxial magneto-mechanical model. This method has been followed by several authors in the past years [24–27]. These equivalent stresses have been compared recently [29], we only briefly recall their definition hereafter.

Schneider and Richardson [24] proposed an equivalent stress for biaxial loadings applied to sheet specimen (Eq. (21)). It introduces σ_1 and σ_2 the eigenvalues of the stress tensor in the sheet plane, the magnetic field being applied along direction 1. Equibiaxial tension or compression is supposed to have no effect on the magnetic behavior:

$$\sigma_{eq}^{SR} = \sigma_1 - \sigma_2 \tag{21}$$

On the basis of experimental biaxial measurements, Kashiwaya [25] proposed a slightly different definition (Eq. (22)), σ_1 being the eigenstress parallel to the magnetic field and σ_{max} the maximum eigenvalue of the stress tensor. K is a constant that can be adjusted for a better fitting of experimental results. This equivalent stress is always negative. Tensile stress or equi-biaxial compression is supposed to have no effect on the magnetic behavior:

$$\sigma_{eq}^K = K(\sigma_1 - \sigma_{max}) \tag{22}$$

Based on biaxial measurement results [30], Sablik et al. [26] proposed another definition (Eq. (23)). σ_1 and σ_2 are still the eigenvalues of the stress tensor in the sheet plane, the magnetic field being applied along direction 1. It can be noticed that the definition is discontinuous for $\sigma_1 = 0$ and that in the case of a uniaxial loading the equivalent stress does not reduce to the applied stress:

$$\begin{cases} \sigma_{eq}^S = \frac{1}{3}(2\sigma_1 - \sigma_2) & \text{if } \sigma_1 < 0 \\ \sigma_{eq}^S = \frac{1}{3}(\sigma_1 - 2\sigma_2) & \text{if } \sigma_1 \geq 0 \end{cases} \tag{23}$$

Pearson et al. [27], under similar assumptions and using the notations of Schneider and Richardson's proposal, make use of a function $g(\sigma)$ (Eq. (24)). The equivalent stress is not explicitly defined but using $\sigma_{eq} = \sigma_1$ for a uniaxial loading, an explicit expression can be obtained. Although more accurate than the previous proposals, this definition does not address the case of a magnetic field not aligned along an eigendirection of the stress tensor. Moreover this criterion is highly material and specimen dependent, and complicated to implement due to the large number of parameters to identify:

$$g(\sigma) = k\sqrt{\sigma_1^2 + \sigma_2^2} + \sum_{n=1}^6 a_n (f\sigma_1 - \sigma_2)^n + b\sigma_1 \tag{24}$$

In the configuration corresponding to a biaxial stress with eigenvalues in directions 1 and 2, and a magnetic field applied

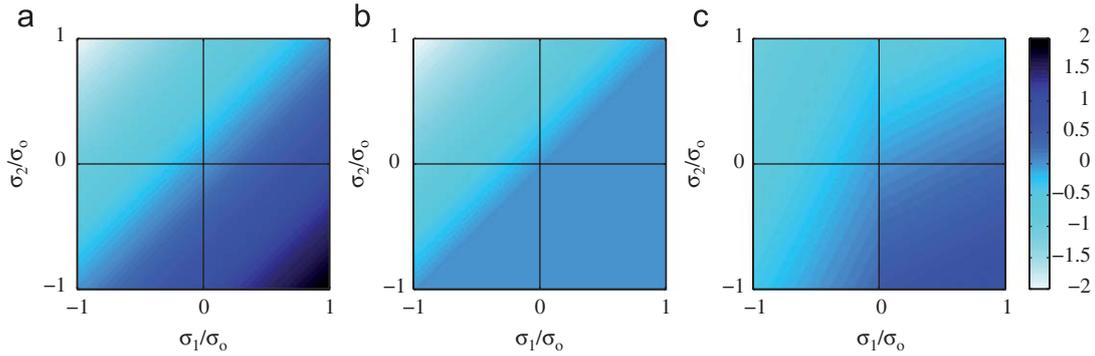


Fig. 7. Adimensional iso-values of several equivalent stress proposals (σ_{eq}/σ_0): (a) Schneider and Richardson [24]; (b) Kashiwaya [25] (with $K=1$); (c) Sablik et al. [26].

along direction 1, the different equivalent stress proposals have been compared in Fig. 7 (the adimensional ratio σ_{eq}/σ_0 is plotted as a function of σ_1/σ_0 and σ_2/σ_0 . Pearson et al. proposal has not been reported due to the complexity of the parameter identification.

The former proposals for an equivalent stress exhibit strong limitations: the mechanical loading is restricted to biaxial stress, the magnetic field is necessarily applied along an eigendirection of the stress tensor and they are restricted to isotropic materials. The definition of a more general equivalent stress is requisite considering the much more complex range of combined magnetic and mechanical loadings that can be encountered in practical applications (see Section 1). The multiscale model in its continuous form gives directions of investigation. Homogeneous field and stress conditions over the grain are considered.

4. Equivalent stress definition from an equivalence in magneto-elastic energy

The following proposal of equivalent stress for isotropic materials based on an equivalence in magnetoelastic energy has been recently published [28].

Let us consider the definition of the magneto-elastic energy at the domain scale (Eq. (9)). An integration of the magneto-elastic energy over the volume leads to the macroscopic magneto-elastic energy so that

$$W^\sigma = - \int_{\alpha} f_{\alpha} \boldsymbol{\sigma}_g : \boldsymbol{\varepsilon}_{\alpha}^{\mu} d\alpha \quad (25)$$

Since homogeneous stress condition ($\boldsymbol{\sigma}_g = \boldsymbol{\sigma}$) is assumed over the volume and using Eq. (20) we get

$$W^\sigma = - \int_{\alpha} f_{\alpha} \boldsymbol{\sigma} : \boldsymbol{\varepsilon}_{\alpha}^{\mu} d\alpha = - \boldsymbol{\sigma} : \int_{\alpha} f_{\alpha} \boldsymbol{\varepsilon}_{\alpha}^{\mu} d\alpha = - \boldsymbol{\sigma} : \boldsymbol{\varepsilon}^{\mu} \quad (26)$$

$\boldsymbol{\sigma}$ is taken as multiaxial (six independent terms) in the orthonormal coordinate system ($\vec{e}_x, \vec{e}_y, \vec{e}_z$):

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{pmatrix} \quad (27)$$

Hypotheses must now be given on the macroscopic magnetostriction strain. The major hypothesis is to consider macroscopic magnetostriction as independent from stress, neglecting the so-called ΔE effect [54]. $\boldsymbol{\varepsilon}^{\mu}$ is then only linked to the magnetic field strength and direction \vec{H} . Moreover magnetostriction is considered as isovolumetric (following a usual hypothesis for standard magnetic materials). Complementary hypotheses concern the symmetries of the material.

4.1. Isotropic material

In the case of an isotropic material, and considering a magnetic field applied along direction \vec{e}_z , $\boldsymbol{\varepsilon}^{\mu}$ is given by

$$\boldsymbol{\varepsilon}^{\mu} = \begin{pmatrix} -\frac{1}{2}\lambda(H) & 0 & 0 \\ 0 & -\frac{1}{2}\lambda(H) & 0 \\ 0 & 0 & \lambda(H) \end{pmatrix} \quad (28)$$

where $\lambda(H)$ is the deformation measured along the magnetic field direction. The magnetoelastic energy is given by

$$W^\sigma = \frac{1}{2} \sigma_{xx} \lambda(H) + \frac{1}{2} \sigma_{yy} \lambda(H) - \sigma_{zz} \lambda(H) = -\lambda(H) \left(\frac{3}{2} \sigma_{zz} - \frac{1}{2} (\sigma_{xx} + \sigma_{yy} + \sigma_{zz}) \right) \quad (29)$$

In order to get a definition independent from the chosen coordinate system, the stress component in the direction of the magnetic field is written as $\sigma_{zz} = {}^t \vec{h} \boldsymbol{\sigma} \vec{h}$ where \vec{h} denotes the direction of the applied field and ${}^t \vec{h}$ the transpose of \vec{h} . The frame associated to the magnetic field is $(\vec{h}, \vec{t}_1, \vec{t}_2)$. We also recognize the trace of the stress tensor in Eq. (29): $\text{tr}(\boldsymbol{\sigma}) = \sigma_{xx} + \sigma_{yy} + \sigma_{zz}$. The expression for the magnetoelastic energy is finally written, for any stress tensor $\boldsymbol{\sigma}$:

$$W^\sigma = -\lambda(H) \left(\frac{3}{2} {}^t \vec{h} \boldsymbol{\sigma} \vec{h} - \frac{1}{2} \text{tr}(\boldsymbol{\sigma}) \right) \quad (30)$$

Let now consider a uniaxial stress σ_u applied in the direction parallel to the magnetic field \vec{h} :

$$\boldsymbol{\sigma} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_u \end{pmatrix}_{(\vec{t}_1, \vec{t}_2, \vec{h})} \quad (31)$$

The corresponding magnetoelastic energy, according to Eq. (26) is simply

$$W_u^\sigma = -\lambda(H) \sigma_u \quad (32)$$

If we assume that the same magnetoelastic energy leads to the same magnetic behavior, the equivalent stress σ_{eq} is corresponding to the component σ_u once Eqs. (30) and (32) are considered equivalent. The following expression for the equivalent stress is finally obtained as

$$\sigma_{eq} = \sigma_u = \frac{3}{2} {}^t \vec{h} \boldsymbol{\sigma} \vec{h} - \frac{1}{2} \text{tr}(\boldsymbol{\sigma}) = \frac{3}{2} {}^t \vec{h} \boldsymbol{s} \vec{h} \quad (33)$$

where \boldsymbol{s} is the deviatoric part of the stress tensor $\boldsymbol{\sigma}$ ($\boldsymbol{s} = \boldsymbol{\sigma} - \frac{1}{3} \text{tr}(\boldsymbol{\sigma}) \mathbf{I}$, with \mathbf{I} the identity second order tensor).

4.2. Orthotropic material

In the case of an orthotropic material, $\boldsymbol{\varepsilon}^{\mu}$ has not a unique definition: it depends on the direction of the magnetic field with respect to the orthotropic frame. We consider first a magnetic field applied along a direction of orthotropy \vec{e}_z ; $\boldsymbol{\varepsilon}^{\mu}$ is diagonal in

the frame of orthotropy so that

$$\varepsilon^\mu = \begin{pmatrix} -\frac{(1+\beta)}{2}\lambda(H) & 0 & 0 \\ 0 & -\frac{(1-\beta)}{2}\lambda(H) & 0 \\ 0 & 0 & \lambda(H) \end{pmatrix}_{(\vec{e}_x, \vec{e}_y, \vec{e}_z)} \quad (34)$$

where coefficient β indicates a degree of orthotropy. Since orthotropy and magnetic field frame are coincident $((\vec{e}_x, \vec{e}_y, \vec{e}_z) = (\vec{t}_1, \vec{t}_2, \vec{h}))$, β can formally be defined as a ratio:

$$\beta = \frac{t_2^\mu \varepsilon^\mu t_2 - t_1^\mu \varepsilon^\mu t_1}{t_2^\mu \varepsilon^\mu t_2 + t_1^\mu \varepsilon^\mu t_1} \quad (35)$$

When $\beta = 0$, the isotropic situation is recovered. $\beta = 1$ condition is frequently encountered for sheet form specimens where the material exhibits a very low deformation through the thickness (non-oriented silicon-iron, iron-cobalt) due to a specific domain configuration [55]. Grain-oriented silicon-iron sheets exhibit on the other hand a value for β higher than 1 when the material is magnetized along the transversal direction for example [56]. The magnetoelastic energy is given by

$$W^\sigma = \frac{(1+\beta)}{2} \sigma_{xx} \lambda(H) + \frac{(1-\beta)}{2} \sigma_{yy} \lambda(H) - \sigma_{zz} \lambda(H) \quad (36)$$

Following the same mathematical developments than in the previous paragraph, we get a more complex definition of the equivalent stress:

$$\sigma_{eq} = \frac{3}{2} \vec{h} \mathbf{s} \vec{h} + \frac{\beta}{2} (t_2^\mu \mathbf{s} t_2 - t_1^\mu \mathbf{s} t_1) \quad (37)$$

This definition is limited to the cases where magnetic field is aligned with an orthotropic direction. This criterion is for example not applicable when the field is applied in a direction between the rolling or transverse direction for GO silicon steels. It is not applicable to the single crystalline situation when a direction other than $\langle 100 \rangle$ or $\langle 110 \rangle$ is considered.

We consider now that the magnetic field is not applied along a direction of orthotropy, ε^μ is nevertheless diagonal in its own eigenframe $(\vec{e}_I, \vec{e}_{II}, \vec{e}_{III})$.¹ Eigenvalues are noted ε_I^μ , ε_{II}^μ and ε_{III}^μ , and verify $\varepsilon_I^\mu + \varepsilon_{II}^\mu + \varepsilon_{III}^\mu = 0$. Parameters β_1 and β_2 are introduced so that $\varepsilon_I^\mu = \beta_1 \lambda(H)$, $\varepsilon_{II}^\mu = \beta_2 \lambda(H)$ and $\varepsilon_{III}^\mu = -(\beta_1 + \beta_2) \lambda(H)$. The magnetostriction strain tensor is finally

$$\varepsilon^\mu = \begin{pmatrix} \beta_1 \lambda(H) & 0 & 0 \\ 0 & \beta_2 \lambda(H) & 0 \\ 0 & 0 & -(\beta_1 + \beta_2) \lambda(H) \end{pmatrix}_{(\vec{e}_I, \vec{e}_{II}, \vec{e}_{III})} \quad (38)$$

The magnetic field is not applied along \vec{e}_I , \vec{e}_{II} or \vec{e}_{III} . It is written in the coordinate system using two spherical angles θ and ϕ :

$$\vec{h} = \begin{pmatrix} \cos \phi \sin \theta \\ \sin \phi \sin \theta \\ \cos \theta \end{pmatrix}_{(\vec{e}_I, \vec{e}_{II}, \vec{e}_{III})} \quad (39)$$

Considering a stress tensor written in the eigenframe, the magnetoelastic energy is given by

$$W^\sigma = -\beta_1 \sigma_{xx} \lambda(H) - \beta_2 \sigma_{yy} \lambda(H) + (\beta_1 + \beta_2) \sigma_{zz} \lambda(H) \quad (40)$$

The uniaxial stress σ_u considered for the expression of equivalent stress is applied along the magnetic field vector \vec{h} . The corresponding magnetoelastic energy is

$$W_u^\sigma = (-\beta_1 \cos^2 \phi \sin^2 \theta - \beta_2 \sin^2 \phi \sin^2 \theta + (\beta_1 + \beta_2) \cos^2 \theta) \sigma_u \lambda(H) \quad (41)$$

¹ The eigenframe is not rotating since magnetization rotation is not considered in the magnetization process. If magnetization rotation occurs, the frame is moving and a general formulation of equivalent stress is no more reachable.

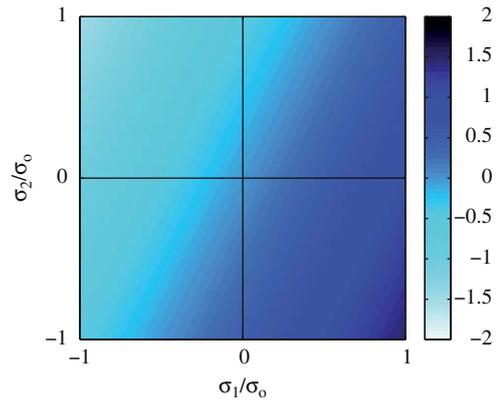


Fig. 8. Adimensional iso-values of deviatoric equivalent stress (σ_{eq}/σ_0).

A final form of equivalent stress is finally obtained, based on an equivalence in magneto-elastic energy and considering no magnetization rotation.

$$\sigma_{eq} = \frac{-\beta_1 \sigma_{xx} - \beta_2 \sigma_{yy} + (\beta_1 + \beta_2) \sigma_{zz}}{-\beta_1 \cos^2 \phi \sin^2 \theta - \beta_2 \sin^2 \phi \sin^2 \theta + (\beta_1 + \beta_2) \cos^2 \theta} \quad (42)$$

The equivalent stress can be written as a function of the deviatoric stress tensor:

$$\sigma_{eq} = \frac{-\beta_1^t \vec{e}_I \mathbf{s} \vec{e}_I - \beta_2^t \vec{e}_{II} \mathbf{s} \vec{e}_{II} + (\beta_1 + \beta_2)^t \vec{e}_{III} \mathbf{s} \vec{e}_{III}}{-\beta_1 \cos^2 \phi \sin^2 \theta - \beta_2 \sin^2 \phi \sin^2 \theta + (\beta_1 + \beta_2) \cos^2 \theta} \quad (43)$$

This final form is more general than the previous one. The major difficulty is to define the eigenframe of magnetostriction with respect to the directions of magnetic and mechanical loadings.

4.3. Discussion about the deviatoric equivalent stress—*isotropic media*

We are considering the simplest form of equivalent stress (Eq. (33)) defined by an equivalence in magneto-elastic energy for an isotropic material. The following properties can be highlighted:

- in the case of a uniaxial stress applied in the direction of the magnetic field, the equivalent stress is the applied stress;
- the definition can be applied to a fully multiaxial mechanical loading, not only biaxial;
- any orientation of the stress tensor with respect to the magnetic field can be considered;
- a hydrostatic pressure leads to an equivalent stress equal to zero, in agreement with the noneffect of hydrostatic pressure on magnetic behavior.

In the case of a biaxial mechanical loading (σ_1, σ_2) , with \vec{h} aligned with direction \vec{e}_I , the equivalent stress is given by

$$\sigma_{eq} = \sigma_1 - \frac{\sigma_2}{2} \quad (44)$$

Results have been plotted in Fig. 8 (the adimensional ratio σ_{eq}/σ_0 is plotted as a function of σ_1/σ_0 and σ_2/σ_0). Comparison to experimental data will be presented in the final section.

5. Equivalent stress definition from an equivalence in magnetization

We look back to the original definition of the equivalent stress: it is the uniaxial stress that leads to a magnetic behavior corresponding to the magnetic behavior of the material submitted to the multiaxial stress. The magnetic field being imposed, the

magnetic response is the magnetization. The major difficulty is that magnetization depends on stress and magnetic field: effects cannot be easily separated. We propose here an approach using a surrogate multiscale model, only focused on the magneto-mechanical behavior of *one grain*, defined as an assembly of magnetic domains.

5.1. General formulation

Eq. (19) recalled here after (Eq. (45)) gives the definition of the macroscopic magnetization of the polycrystal using the surrogate multiscale model: isotropic material is seen as an assembly of magnetic domains equally distributed in the volume. The definition of the volumetric fraction follows the same approach (Eq. (46)):

$$\vec{M} = \int_{\alpha} f_{\alpha} \vec{M}_{\alpha} d\alpha \quad (45)$$

$$f_{\alpha} = \frac{\exp(-A_s \cdot W_{\alpha})}{\int_{\alpha} \exp(-A_s \cdot W_{\alpha})} \quad (46)$$

We suppose now that only the energetical terms associated to the loading have to be taken into account. This assumption means neglecting the role of magnetocrystalline anisotropy energy in the magnetization process. Moreover stress and magnetic field are supposed to be uniform within the material. We obtain the following definition of the volumetric fraction of a domain α :

$$f_{\alpha} = \frac{\exp(A_s \sigma : \varepsilon_{\alpha}^{\mu} + A_s \mu_0 \vec{H} \cdot \vec{M}_{\alpha})}{\int_{\alpha} \exp(A_s \sigma : \varepsilon_{\alpha}^{\mu} + A_s \mu_0 \vec{H} \cdot \vec{M}_{\alpha})} \quad (47)$$

Magnetization under multiaxial stress condition σ is given by

$$\vec{M} = \int_{\alpha} \frac{\exp(A_s \sigma : \varepsilon_{\alpha}^{\mu} + A_s \mu_0 \vec{H} \cdot \vec{M}_{\alpha})}{\int_{\alpha} \exp(A_s \sigma : \varepsilon_{\alpha}^{\mu} + A_s \mu_0 \vec{H} \cdot \vec{M}_{\alpha})} \vec{M}_{\alpha} d\alpha \quad (48)$$

It has to be compared to the magnetization obtained under uniaxial stress condition σ_u applied in the direction of the magnetic field:

$$\vec{M}_u = \int_{\alpha} \frac{\exp(A_s \sigma_u : \varepsilon_{\alpha}^{\mu} + A_s \mu_0 \vec{H} \cdot \vec{M}_{\alpha})}{\int_{\alpha} \exp(A_s \sigma_u : \varepsilon_{\alpha}^{\mu} + A_s \mu_0 \vec{H} \cdot \vec{M}_{\alpha})} \vec{M}_{\alpha} d\alpha \quad (49)$$

The uniaxial stress σ_u is finally corresponding to the equivalent stress σ_{eq} when $\vec{M} = \vec{M}_u$ so that σ_{eq} is a solution of the following equation:

$$\int_{\alpha} \frac{\exp(A_s \sigma_{eq} : \varepsilon_{\alpha}^{\mu} + A_s \mu_0 \vec{H} \cdot \vec{M}_{\alpha})}{\int_{\alpha} \exp(A_s \sigma_{eq} : \varepsilon_{\alpha}^{\mu} + A_s \mu_0 \vec{H} \cdot \vec{M}_{\alpha})} \vec{M}_{\alpha} d\alpha = \int_{\alpha} \frac{\exp(A_s \sigma : \varepsilon_{\alpha}^{\mu} + A_s \mu_0 \vec{H} \cdot \vec{M}_{\alpha})}{\int_{\alpha} \exp(A_s \sigma : \varepsilon_{\alpha}^{\mu} + A_s \mu_0 \vec{H} \cdot \vec{M}_{\alpha})} \vec{M}_{\alpha} d\alpha \quad (50)$$

with $\sigma_{eq} = {}^t \vec{h} \sigma_{eq} \vec{h}$.

This equation is rather difficult to solve. Some previous works showed that an integration can be analytically done considering magnetic field or stress as zero [21,54]. A combination of the two loadings is necessary in the present case. A numerical resolution would be possible but our objective here is to propose an analytical expression for the equivalent stress. We then have to simplify Eq. (50) in order to identify σ_{eq} .

5.2. Multidomain model and application to the definition of the equivalent stress

The idea is to replace the volume integral by a discrete sum over a finite number of domains [57]. The domains must be equally distributed. The first admissible distribution is a cubic distribution (first terms of the spherical decomposition), where only six domain

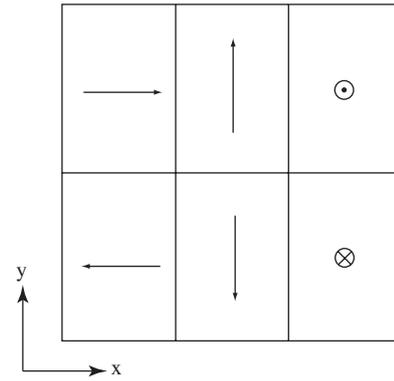


Fig. 9. Domain assembly representative for the material submitted to magnetic field and stress.

families are considered with magnetization along the six $\langle 100 \rangle$ axes of an elementary cube (Fig. 9) in $(\vec{x}, \vec{y}, \vec{z})$ coordinate system. The coordinate system is supposed to be defined from the direction of the applied field so that: $(\vec{x}, \vec{y}, \vec{z}) = (\vec{h}, \vec{t}_1, \vec{t}_2)$. This structure is simultaneously submitted to a magnetic field \vec{H} (Eq. (51)) and to a multiaxial stress tensor σ (Eq. (52)):

$$\vec{H} = H \vec{x} \quad (51)$$

$$\sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{pmatrix}_{xyz} \quad (52)$$

The definition of the magnetostriction strain tensor differs from the definition generally used for a domain because the average behavior has to be in accordance with the behavior of the isotropic material that this simple assembly of “domains” is supposed to model. Considering no rotation mechanism, the definition of the magnetostriction strain tensor in the domain frame (DF) is given by

$$\varepsilon_{\alpha}^{\mu} = \begin{pmatrix} \lambda_m & 0 & 0 \\ 0 & -\frac{1}{2}\lambda_m & 0 \\ 0 & 0 & -\frac{1}{2}\lambda_m \end{pmatrix}_{DF} \quad (53)$$

λ_m is the maximum magnetostriction that can be reached by the isotropic material. This parameter can be identified from experimental measurements, but it can also be defined from the value of the single crystal magnetostriction coefficient λ_{100} or λ_{111} . In the previous work [54] it has been shown that the definition of λ_m depends on the material crystalline symmetry: $\lambda_m = 2/5 \lambda_{100} \cdot k_a$ for positive magnetocrystalline constant materials (such as iron) and $\lambda_m = 3/5 \lambda_{111} \cdot k_b$ for negative magnetocrystalline constant materials (such as nickel). k_a and k_b depend on the elastic properties of the single crystal [21] and on the hypotheses chosen for the description of the material. For instance, if we choose uniform stress (Reuss) hypotheses, we have $k_a = k_b = 1$, and if we choose uniform strain (Voigt) hypotheses, we have $k_a = 5\mu_a / (2\mu_a + 3\mu_b)$ and $k_b = 5\mu_b / (2\mu_a + 3\mu_b)$, μ_a and μ_b being the cubic shear moduli of the single crystal. For the sake of simplicity, we will choose $k_a = k_b = 1$, in further numerical applications.

The equivalent stress must now verify a discrete version of Eq. (50) so that

$$\sum_{\alpha=1..6} \frac{\exp(A_s \sigma_{eq} : \varepsilon_{\alpha}^{\mu} + A_s \mu_0 \vec{H} \cdot \vec{M}_{\alpha})}{\sum_{\alpha=1..6} \exp(A_s \sigma_{eq} : \varepsilon_{\alpha}^{\mu} + A_s \mu_0 \vec{H} \cdot \vec{M}_{\alpha})} \vec{M}_{\alpha} = \sum_{\alpha=1..6} \frac{\exp(A_s \sigma : \varepsilon_{\alpha}^{\mu} + A_s \mu_0 \vec{H} \cdot \vec{M}_{\alpha})}{\sum_{\alpha=1..6} \exp(A_s \sigma : \varepsilon_{\alpha}^{\mu} + A_s \mu_0 \vec{H} \cdot \vec{M}_{\alpha})} \vec{M}_{\alpha} \quad (54)$$

The potential energy of a domain family is now

$$W_\alpha = -\mu_0 \vec{H} \cdot \vec{M}_\alpha - \sigma : \varepsilon_\alpha^{\prime\mu} \quad (55)$$

This energy term can be written for each of the six domain families submitted to the multiaxial stress:

$$W_1 = -\mu_0 H \cdot M_s - \sigma_{xx} \cdot \lambda_m + \sigma_{yy} \cdot \frac{\lambda_m}{2} + \sigma_{zz} \cdot \frac{\lambda_m}{2}$$

$$W_2 = \mu_0 H \cdot M_s - \sigma_{xx} \cdot \lambda_m + \sigma_{yy} \cdot \frac{\lambda_m}{2} + \sigma_{zz} \cdot \frac{\lambda_m}{2}$$

$$W_3 = W_4 = -\sigma_{yy} \cdot \lambda_m + (\sigma_{xx} + \sigma_{zz}) \cdot \frac{\lambda_m}{2}$$

$$W_5 = W_6 = -\sigma_{zz} \cdot \lambda_m + (\sigma_{xx} + \sigma_{yy}) \cdot \frac{\lambda_m}{2}$$

It can be noticed that the shear stresses σ_{xy} , σ_{xz} and σ_{yz} do not appear in the result. After calculation, the right hand term of Eq. (54) becomes

$$M_s \frac{\sinh(\mu_0 M_s A_s H) \exp\left(A_s \lambda_m \left(\sigma_{xx} - \frac{\sigma_{yy}}{2} - \frac{\sigma_{zz}}{2}\right)\right)}{A+B+C} \vec{x} \quad (56)$$

with

$$A = \cosh(\mu_0 M_s A_s H) \exp\left(A_s \lambda_m \left(\sigma_{xx} - \frac{\sigma_{yy}}{2} - \frac{\sigma_{zz}}{2}\right)\right)$$

$$B = \exp\left(A_s \lambda_m \left(\sigma_{yy} - \frac{\sigma_{xx}}{2} - \frac{\sigma_{zz}}{2}\right)\right)$$

$$C = \exp\left(A_s \lambda_m \left(\sigma_{zz} - \frac{\sigma_{xx}}{2} - \frac{\sigma_{yy}}{2}\right)\right)$$

Considering the equivalent uniaxial stress, the potential energy of a domain family is

$$W_\alpha = -\mu_0 \vec{H} \cdot \vec{M}_\alpha - \sigma_{eq} : \varepsilon_\alpha^{\prime\mu} \quad (57)$$

We obtain the following energy terms for the six domain families:

$$W_1 = -\mu_0 H \cdot M_s - \sigma_{eq} \cdot \lambda_m$$

$$W_2 = \mu_0 H \cdot M_s - \sigma_{eq} \cdot \lambda_m$$

$$W_3 = W_4 = \sigma_{eq} \cdot \frac{\lambda_m}{2}$$

$$W_5 = W_6 = \sigma_{eq} \cdot \frac{\lambda_m}{2}$$

After calculation, the left hand term of Eq. (54) becomes

$$M_s \frac{\sinh(\mu_0 M_s A_s H) \exp(A_s \lambda_m \sigma_{eq})}{\cosh(\mu_0 M_s A_s H) \exp(A_s \lambda_m \sigma_{eq}) + 2 \exp(-1/2 A_s \lambda_m \sigma_{eq})} \vec{x} \quad (58)$$

Expressions (56) and (58) can be rewritten respectively in

$$M_s \frac{\sinh(\mu_0 M_s A_s H)}{\cosh(\mu_0 M_s A_s H) + \frac{\exp\left(A_s \lambda_m \left(\sigma_{yy} - \frac{\sigma_{xx}}{2} - \frac{\sigma_{zz}}{2}\right)\right) + \exp\left(A_s \lambda_m \left(\sigma_{zz} - \frac{\sigma_{xx}}{2} - \frac{\sigma_{yy}}{2}\right)\right)}{\exp\left(A_s \lambda_m \left(\sigma_{xx} - \frac{\sigma_{yy}}{2} - \frac{\sigma_{zz}}{2}\right)\right)} \vec{x} \quad (59)$$

and

$$M_s \frac{\sinh(\mu_0 M_s A_s H)}{\cosh(\mu_0 M_s A_s H) + \frac{2 \exp\left(-\frac{1}{2} A_s \lambda_m \sigma_u\right)}{\exp\left(A_s \lambda_m \sigma_u\right)}} \vec{x} \quad (60)$$

σ_{eq} being solution of Eq. (54), the equality of expressions (59) and (60) leads to

$$\exp\left(\frac{3}{2} A_s \lambda_m \sigma_{eq}\right) = \frac{2 \exp\left(A_s \lambda_m \left(\sigma_{xx} - \frac{\sigma_{yy}}{2} - \frac{\sigma_{zz}}{2}\right)\right)}{\exp\left(A_s \lambda_m \left(\sigma_{yy} - \frac{\sigma_{xx}}{2} - \frac{\sigma_{zz}}{2}\right)\right) + \exp\left(A_s \lambda_m \left(\sigma_{zz} - \frac{\sigma_{xx}}{2} - \frac{\sigma_{yy}}{2}\right)\right)} \quad (61)$$

and finally after few calculations

$$\sigma_{eq} = \frac{2}{3 A_s \lambda_m} \ln \left[\frac{2 \exp\left(A_s \lambda_m \left(\sigma_{xx} - \frac{\sigma_{yy}}{2} - \frac{\sigma_{zz}}{2}\right)\right)}{\exp\left(A_s \lambda_m \left(\sigma_{yy} - \frac{\sigma_{xx}}{2} - \frac{\sigma_{zz}}{2}\right)\right) + \exp\left(A_s \lambda_m \left(\sigma_{zz} - \frac{\sigma_{xx}}{2} - \frac{\sigma_{yy}}{2}\right)\right)} \right] \quad (62)$$

This definition can be generalized to any frame $(\vec{h}, \vec{t}_1, \vec{t}_2)$ associated to the magnetic field direction \vec{h} . We note k the product $A_s \lambda_m$ as a material dependent parameter:

$$\sigma_{eq} = {}^t \vec{h} \mathbf{s} \vec{h} - \frac{2}{3k} \ln \left[\frac{\exp\left(\frac{3k}{2} \vec{t}_1 \mathbf{s} \vec{t}_1\right) + \exp\left(\frac{3k}{2} \vec{t}_2 \mathbf{s} \vec{t}_2\right)}{2} \right] \quad (63)$$

Considering the definitions of A_s and λ_m , and using the hypothesis of uniform stress ($k_a = k_b = 1$) we get

$$k = \frac{6 \chi_0 \lambda_{100}}{5 \mu_0 M_s^2} \quad (64)$$

for positive magnetocrystalline anisotropy materials and

$$k = \frac{9 \chi_0 \lambda_{111}}{5 \mu_0 M_s^2} \quad (65)$$

for negative magnetocrystalline anisotropy materials. χ_0 denotes the initial anhysteretic susceptibility of the material, μ_0 the permeability of vacuum and M_s the saturation magnetization of the material. This definition of equivalent stress finally requires to know some relatively usual material parameters and *no supplementary adjusting parameter*. This new expression for the equivalent stress generalizes the deviatoric expression (Eq. (33)) and is applicable to cases of more intense loadings and/or highly magnetostrictive materials. The effect of stress on magnetostriction (ΔE effect) is taken into account. An extension to anisotropic materials is possible following the strategy developed in Section 3.2. This new expression will be referred to as the generalized equivalent stress in the following.

5.3. Discussion about the generalized equivalent stress

The properties that can be highlighted are very similar to the properties of the deviatoric equivalent stress:

- in the case of a uniaxial stress applied in the direction of the magnetic field, the equivalent stress is the applied stress;
- the definition can be applied to a fully multiaxial mechanical loading, not only biaxial;
- any orientation of the stress tensor with respect to the magnetic field can be considered;
- a hydrostatic pressure leads to an equivalent stress equal to zero, in agreement with the noneffect of hydrostatic pressure on magnetic behavior.

In the case of a biaxial mechanical loading with eigenstresses (σ_1, σ_2) , the deviatoric tensor is

$$\mathbf{s} = \begin{pmatrix} \frac{2}{3} \sigma_1 - \frac{1}{3} \sigma_2 & 0 & 0 \\ 0 & \frac{2}{3} \sigma_2 - \frac{1}{3} \sigma_1 & 0 \\ 0 & 0 & -\frac{1}{3} (\sigma_1 + \sigma_2) \end{pmatrix} \quad (66)$$

If \vec{h} is aligned with direction \vec{e}_1 the equivalent stress is given by

$$\sigma_{eq} = \frac{2}{3} \sigma_1 - \frac{1}{3} \sigma_2 - \frac{2}{3k} \ln \left[\frac{\exp(k(\sigma_2 - \frac{1}{2} \sigma_1)) + \exp(k(-\frac{1}{2} \sigma_2 - \frac{1}{2} \sigma_1))}{2} \right] \quad (67)$$

that can be simplified into

$$\sigma_{eq} = \sigma_1 - \frac{2}{3k} \ln \left[\frac{\exp(\frac{3k}{2} \sigma_2) + 1}{2} \right] \quad (68)$$

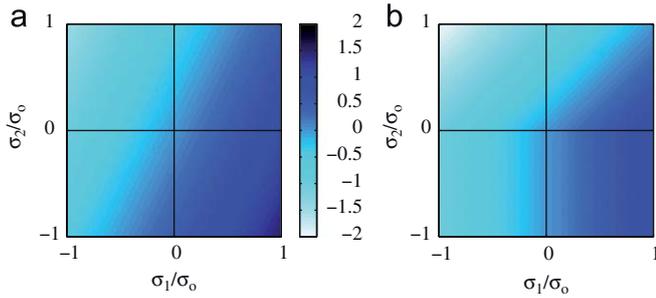


Fig. 10. Adimensional iso-values of generalised equivalent stress (σ_{eq}/σ_0)—for $k > 0$ and (a) $|k\sigma_2| \ll 1$; (b) $|k\sigma_2| \gg 1$.

A direct plot of the generalized equivalent stress is not possible since parameter k is material dependent. We observe that the final expression is more or less complex depending on the stress value along \vec{e}_2 axis:

- for $\sigma_2 = 0$, $\sigma_{eq} = \sigma_1$;
- for $|k\sigma_2| \ll 1$, a Taylor expansion of exponential and logarithm gives

$$\sigma_{eq} \sim \sigma_1 - \frac{2}{3k} \ln \left[\frac{\frac{3k}{2}\sigma_2 + 2}{2} \right] \sim \sigma_1 - \frac{2}{3k} \ln \left[\frac{3k}{4}\sigma_2 + 1 \right] \sim \sigma_1 - \frac{1}{2} \sigma_2$$

reducing to the deviatoric equivalent stress (no assumption is necessary concerning the stress level along \vec{e}_1 axis). This situation is illustrated in Fig. 10a (the adimensional ratio σ_{eq}/σ_0 is plotted as a function of σ_1/σ_0 and σ_2/σ_0). The deviatoric equivalent stress can be seen as a particular case of the generalized equivalent stress;

- for $k\sigma_2 \gg 1$, $\frac{1}{2}$ can be neglected compared to the exponential. We obtain the following estimation:

$$\sigma_{eq} \sim \sigma_1 - \frac{2}{3k} \ln \left[\frac{\exp(\frac{3k}{2}\sigma_2)}{2} \right] \sim \sigma_1 - \sigma_2 + \frac{2\ln 2}{3k}$$

close to Schneider–Richardson proposal. This situation is illustrated in Fig. 10b for positive values of σ_2 (the adimensional ratio σ_{eq}/σ_0 is plotted as a function of σ_1/σ_0 and σ_2/σ_0). The Schneider–Richardson equivalent stress can be seen as a particular case of the generalized equivalent stress;

- for $k\sigma_2 \ll -1$, the exponential becomes negligible. We obtain the following estimation:

$$\sigma_{eq} \sim \sigma_1 - \frac{2}{3k} \ln \left(\frac{1}{2} \right) \sim \sigma_1 + \frac{2\ln 2}{3k}$$

The criterion is now strongly associated to the value of σ_1 . This situation is illustrated by vertical lines in Fig. 10b for negative values of σ_2 (the adimensional ratio σ_{eq}/σ_0 is plotted as a function of σ_1/σ_0 and σ_2/σ_0). This situation was not covered by any criterion of literature;

- intermediate values of $k\sigma_2$ will give a continuous criterion between the previous extremal solutions;
- a change of sign of k (i.e. change of sign of magnetostriction) will give a symmetric plot with respect to \vec{e}_1 axis.

A comparison to experimental data is presented in the next section.

6. Comparison experiments-modeling—magnetic behavior of Fe–Co thin sheet submitted to biaxial mechanical loading

The material used for experiments is a 49%Co–49%Fe–2%V alloy delivered in 0.5 mm thick sheets format (industrial

denomination: AFK 502-R from Imphy Alloys). Cobalt-based alloys are usually known to exhibit a strong saturation magnetization ($M_s = 1.91 \times 10^6$ A/m), that promotes high torque/weight performances for aeronautic equipments. Experiments consist in anhysteretic magnetic measurements carried out under uniaxial and biaxial mechanical stress in homogeneous magnetic and mechanical conditions. A full description of the two set-ups used for measurements can be found in [54] for uniaxial set-up (Fig. 11a) and in [35] for biaxial set-up (Fig. 11b).

6.1. Magnetic and magnetostrictive measurements under uniaxial mechanical loading

The benchmark for magneto-mechanical measurements is detailed in [54]. Measurements carried out are anhysteretic (reversible) magnetic behavior $M(H)$, “parallel” (parallel to the magnetic field direction) and “perpendicular” (perpendicular to the magnetic field direction) magnetostrictive behavior (resp. $\varepsilon_{||}^{\mu}$ and $\varepsilon_{\perp}^{\mu}$).² The applied stress is positive (tension). Fig. 12 shows the evolution of the magnetization curve with respect to the applied stress: we observe a clear increase in susceptibility with increasing stress. Corresponding magnetostrictive behavior has been plotted in Figs. 13a and b. The behavior seems roughly isotropic since perpendicular magnetostriction is negative and about half the amplitude of parallel magnetostriction. Influence of stress is also illustrated: tensile stress progressively saturates the magnetostriction.

Measurements have been carried out on a sample cut along the rolling direction. Measurements along the transverse direction (TD) give similar results. As expected from magnetostrictive measurements, the magneto-mechanical behavior can be considered as isotropic.

6.2. Magnetic measurements under biaxial mechanical loading

These experimental results have already been published in [35]. We recall the main results. Seventeen biaxial stress conditions (σ_1, σ_2) have been tested, for stress level varying from -60 to $+60$ MPa. The magnetic field is applied along direction 1. Mechanical loading can be divided into parallel uniaxial tests ($\sigma_1 \neq 0, \sigma_2 = 0$), orthogonal uniaxial tests ($\sigma_1 = 0, \sigma_2 \neq 0$), equibiaxial tests ($\sigma_1 = \sigma_2$), and shear tests ($\sigma_1 = -\sigma_2$) in order to map the stress plane.

Fig. 14 shows the evolution of the anhysteretic $M(H)$ curve for the parallel uniaxial situation ($\sigma_1 \neq 0, \sigma_2 = 0$). Results are in accordance with measurements carried out using uniaxial set-up: improvement of the magnetization behavior with positive stress. We observe a stronger opposite effect due to compression. Fig. 15 shows the evolution of the anhysteretic $M(H)$ curve for the orthogonal uniaxial situation ($\sigma_1 = 0, \sigma_2 \neq 0$). The effect of the stress level is strongly reduced compared to the previous situation. The sign of the stress is not a dominant parameter since magnetic susceptibility is deteriorated whatever the situation. The deterioration seems a little stronger with positive stress. The equibiaxial situation (Fig. 16) is characterized by a relative insensitivity to stress when the stress value is positive (equitraction). The equibicompaction on the contrary sharply deteriorates the magnetic behavior. The shear situation (Fig. 17) is the worst situation for the magnetic behavior except when σ_1 remains positive. The lowest susceptibility is reached with $\sigma_1 = -60$ MPa and $\sigma_2 = +60$ MPa.

² The readers will find in [54] all experimental details in order to carry out precise measurement of magnetostriction avoiding the difficulties due to the tensile loading.

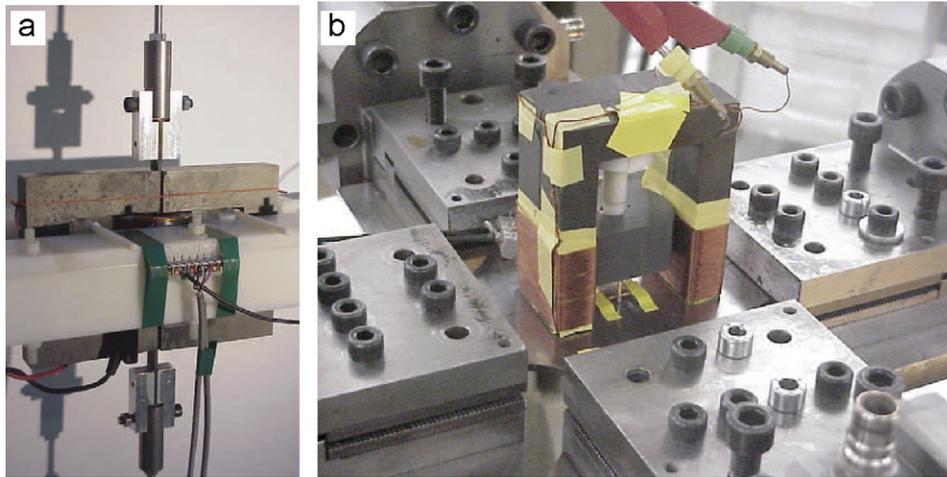


Fig. 11. (a) Benchmark for magnetic and magnetostrictive measurements under tensile stress; (b) general view of the experimental apparatus for magnetic measurement under biaxial stress loading.

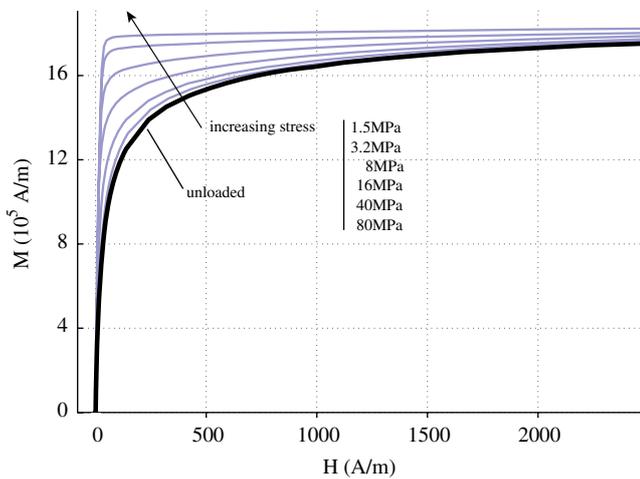


Fig. 12. An hysteretic curve of Fe-Co submitted to uniaxial stress.

A dominant influence of stress level along the axis of magnetic measurement (σ_1) seems to be observed for equibiaxial ($\sigma_1 = \sigma_2$) and shear ($\sigma_1 = -\sigma_2$) conditions; it is especially sensitive when the stress along the orthogonal axis is negative. This analysis also applies for experiments carried out under uniaxial stress. We draw the evolution of the secant susceptibility $\chi = M/H$ in the stress plane in order to illustrate the main trends: $\chi(\sigma_1, \sigma_2)$. Fig. 18a and b plot χ for $H = 250$ and 2500 A/m respectively. The level of magnetic field does not seem a determinant factor. The tension-compression asymmetry is easily perceptible on both graphs. The figures show that a bitraction hardly changes the susceptibility while a bicompression can divide it by two. A compression in a direction perpendicular to the magnetic field has a weak effect, while a tension notably decreases the susceptibility. The lowest values of χ are reached in the upper left side of the graph, corresponding to the shear situation with negative σ_1 .

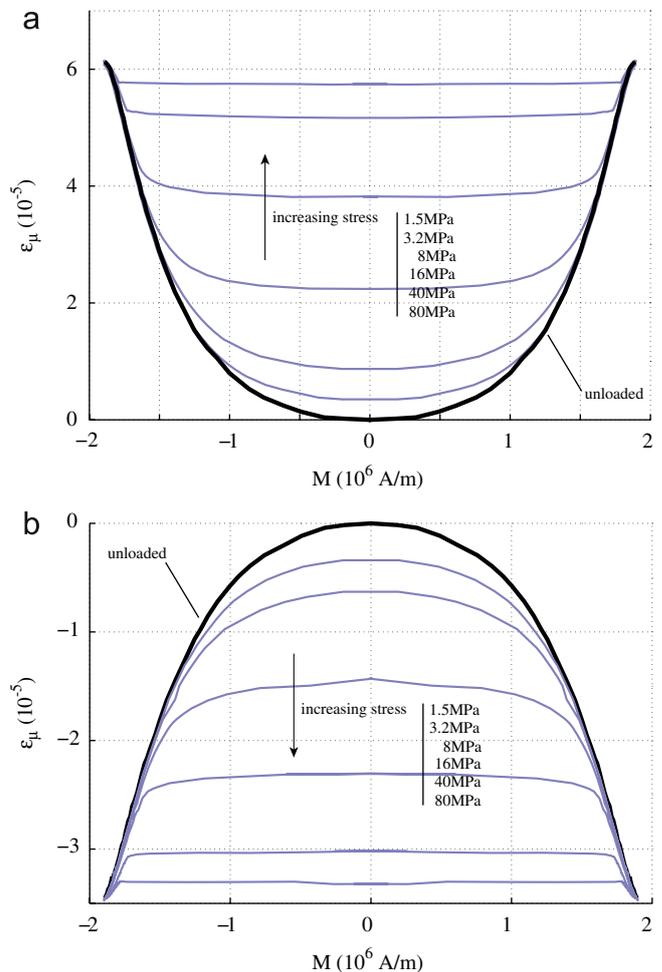


Fig. 13. An hysteretic magnetostriction curves of Fe-Co submitted to uniaxial stress. (a) Parallel magnetostriction, (b) perpendicular magnetostriction.

6.3. Equivalent stresses validation by comparison to biaxial experimental data

In this section, a validation of the several equivalent stress proposals is proposed. Kashiwaya (K), Schneider and Richardson (SR), Sablik et al. (S), deviatoric (d) and generalized (g) equivalent

stress proposals have been detailed in sections 2.2, 3.1 and 4.2. The validation is based on a comparison to biaxial magneto-mechanical measurements. The loading configuration consists in a biaxial stress tensor σ superimposed to a magnetic field \vec{H} aligned in the direction parallel to the first principal stress. This

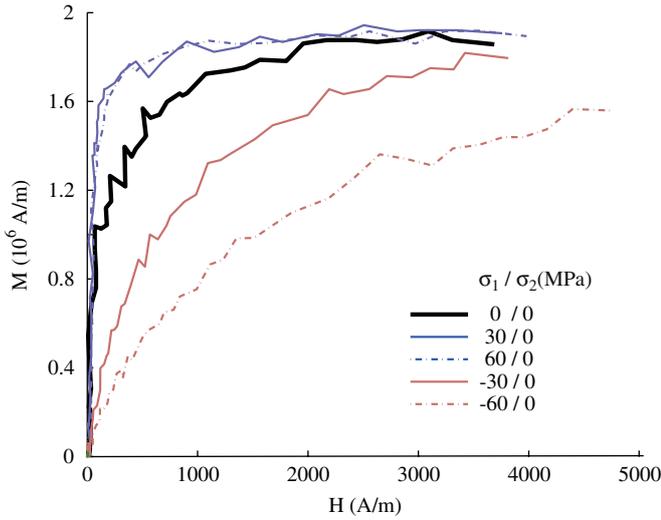


Fig. 14. Anhyseretic $M(H)$ behavior under uniaxial stress: $\sigma \parallel \vec{H}$.

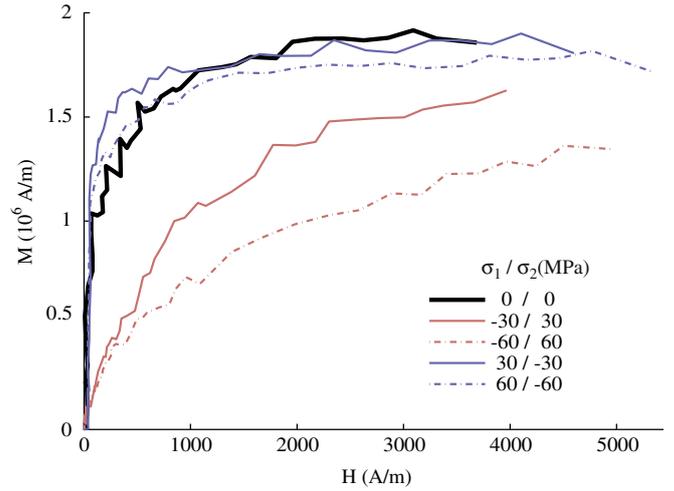


Fig. 17. Anhyseretic $M(H)$ behavior under shearing stress: $\sigma_1 = -\sigma_2$.

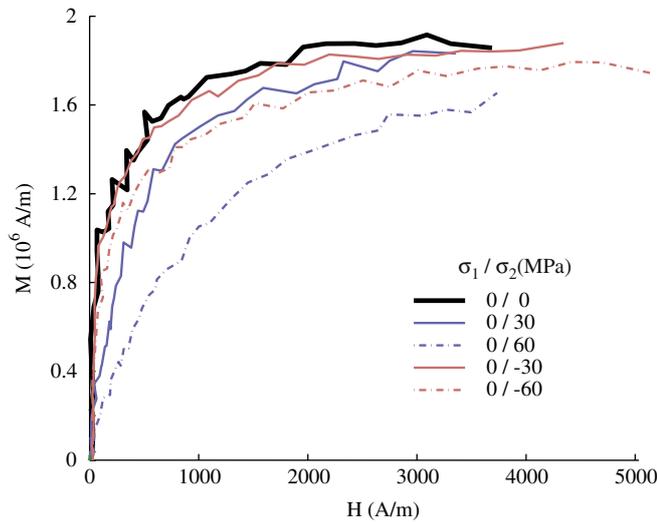


Fig. 15. Anhyseretic $M(H)$ behavior under uniaxial stress: $\sigma \perp \vec{H}$.

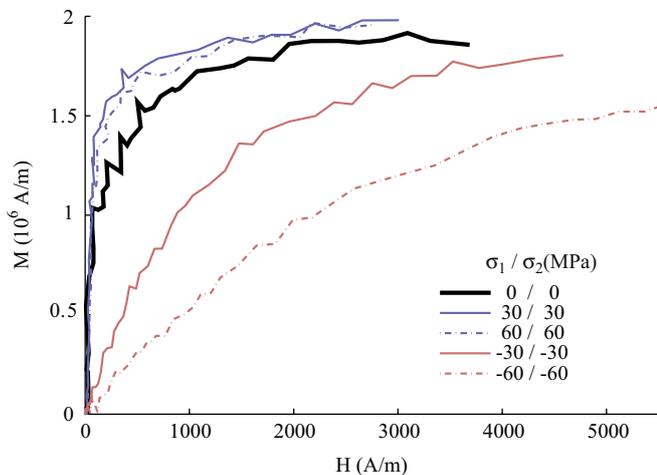


Fig. 16. Anhyseretic $M(H)$ behavior under equibiaxial stress: $\sigma_1 = \sigma_2$.

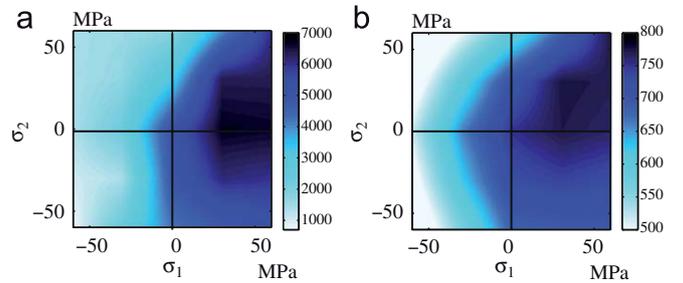


Fig. 18. Normalized susceptibility (experimental results) – $H=250$ A/m (a) $H=2500$ A/m (b) – $\vec{H} \parallel \vec{I}$.

configuration is given by

$$\sigma = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix} \text{ and } \vec{H} = \begin{pmatrix} H \\ 0 \end{pmatrix} \quad (69)$$

The magnetic property of interest for this comparison is the magnetic susceptibility under stress. The validation process is based on four steps:

1. The magnetic susceptibility under uniaxial stress needs to be identified first. It has been collected from various sources (Sections 5.1 and 5.2, and Ref. [50]) for samples exhibiting the same composition. These different results are very consistent. They have been plotted in Figs. 19 and 20 respectively for an applied magnetic field of $H=250$ and 2500 A/m. It is reminded that the magnetic field H is applied in the direction parallel to the – uniaxial – applied stress σ . For modeling purpose, the evolution of the susceptibility as a function of stress has been interpolated, plotted as a full line in Figs. 19 and 20. The interpolation function $\chi = p(\sigma)$ will be used in the following steps.
2. The predicted susceptibility under biaxial stress is then calculated. For K, SR and S and deviatoric equivalent stresses, formulas (21)–(23) and (33) are applied directly. In the case of the generalized equivalent stress, some material parameters are necessary to apply Eq. (63). These material parameters can be identified from unloaded measurements of Figs. 12 and 13: $\lambda_m \approx 65 \times 10^{-6}$, $\chi_0 \approx 3000$ and $M_s = 1.91 \times 10^6$ A/m. Once each equivalent stress σ_{eq} is calculated for a given multiaxial stress σ , the corresponding predicted susceptibility χ^p is calculated using the polynomial interpolation: $\chi^p = p(\sigma_{eq})$.

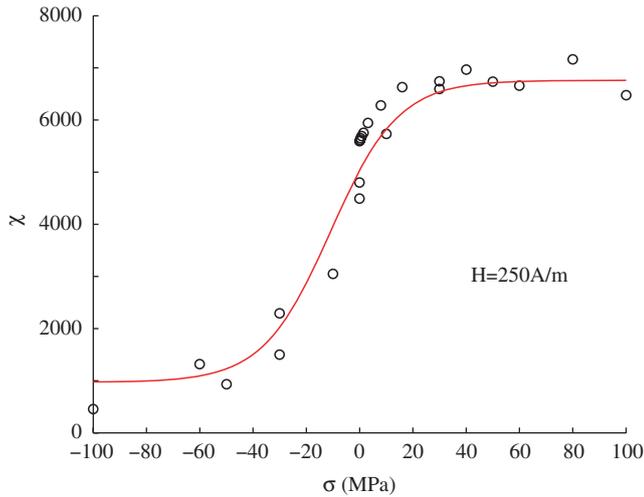


Fig. 19. Magnetic secant susceptibility of iron-cobalt as a function of uniaxial stress for $H=250$ A/m.

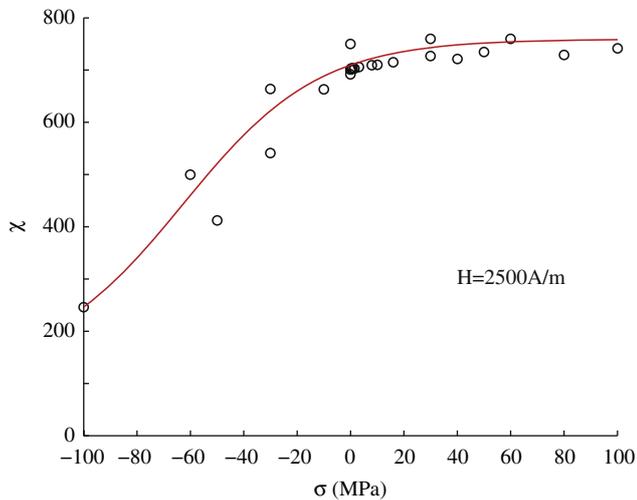


Fig. 20. Magnetic secant susceptibility of iron-cobalt as a function of uniaxial stress for $H=2500$ A/m.

- Experimental results for the susceptibility χ^e under biaxial stress need to be collected. Such biaxial magneto-mechanical measurements have been detailed in Ref. [35] and recalled in Section 5.2. They are reported in Fig. 18, for $H=250$ and 2500 A/m.
- The modeling results of step (2) are compared to independent experimental results of step (3) by the calculation of an error indicator. For each equivalent stress proposal the quantity e (Eq. (70)) is calculated. The lower this indicator, the more accurate the equivalent stress:

$$e = 100 \times \frac{|\chi_p - \chi_e|}{\chi_e} \quad (70)$$

Figs. 21 and 22 plot the error criterion e for K, SR, S, d and g equivalent stresses respectively for $H=250$ A/m and $H=2500$ A/m.

For all criteria except the generalized equivalent stress under high magnetic field, the errors observed are higher in equibiaxial compression than for other loadings. In the case of a stress $(\sigma_1, \sigma_2) = (-60 \text{ MPa}, -60 \text{ MPa})$ and for a magnetic field $H=250$ A/m, the error is up to 635% for K and SR, 371% for S, 238% for d and 117% for g equivalent stress. For reading convenience the error

values have been truncated at 100% to highlight the prediction out of the bicompression area. If the field is increased to $H=2500$ A/m, the error is up to 70% for K and SR, 57% for S, 50% for d and 44% for g equivalent stress. In that case, the generalized stress proposal exhibits higher error levels in the upper right part of the plot, in an area of strong shear stress. The error values have been truncated at 50% for reading convenience. Errors are lower in that latter case because the effect of stress on the magnetic behavior is less sensitive close to magnetic saturation (see for instance Fig. 12).

The deviatoric stress had already been shown to provide a significant improvement of the predicted susceptibility compared to K, SR and S proposals [29]. This result could be reinforced by taking into account the weak anisotropy of the material and choosing the orthotropic version of the deviatoric equivalent stress criterion. Compared to all previous proposals, adding a few material dependent parameters (λ_m , χ_0 and M_s) in the equivalent stress definition, the generalized equivalent stress is a significant improvement for the definition of the magnetic susceptibility under bicompression. These latter equivalent stress definitions make conceivable the use of equivalent stress models even in the case of bi-compressive mechanical loadings. Moreover it is to be recalled that the definition of deviatoric and generalized proposals are applicable for any multiaxial configuration—not only biaxial.

7. Conclusion

In this paper new proposals of equivalent stress for magneto-mechanical behavior have been presented. Equivalent stress is defined as the uniaxial mechanical loading, applied in the direction parallel to the applied magnetic field, that induces the same effect on the magnetic behavior than the corresponding multiaxial stress. The first proposal called “deviatoric equivalent stress” is defined thanks to an equivalence in magnetoelastic energy. The second proposal called “generalized equivalent stress” is defined thanks to an equivalence in magnetization for a given magnetic field. This proposal can be seen as a generalization of the deviatoric equivalent stress. Some stronger simplifications allow on the other hand to build the Schneider and Richardson criterion. The generalized equivalent stress has been constructed after simplification of a full 3D magneto-elastic multiscale model, neglecting the impact of magnetocrystalline energy and thanks to a cubic reconstruction of an idealized isotropic distribution of domains. It must be noticed that the deviatoric definition is easier to use; it may be preferred when $|k\sigma_2| \ll 1$ condition can be assumed (σ_2 is the eigenstress orthogonal to the magnetization direction).

Comparisons to experimental results carried out under biaxial loading show that the generalized equivalent stress gives more accurate predictions than the previous proposals. The criterion reflects the major influence of the stress level along the magnetization axis, especially when stress along the second axis is negative. The deviatoric and generalized equivalent stresses are not restricted to biaxial mechanical loadings and do not require any assumption on the magnetic field direction. The generalized form accounts for the magnetostrictive and magnetic constants of the material and thus reflects the different sensitivity of the magnetic behavior to a mechanical loading depending on the considered material.

This new definition can help to revisit and discuss some previous results of the literature. The results of Pearson et al. [27] and Maurel et al. [33] are accessible and can be re-interpreted. Pearson et al. obtained maps of the relative variation of the coercive field (%) in the biaxial stress plane for a pure iron. Fig. 23a shows the corresponding results. Maurel et al.

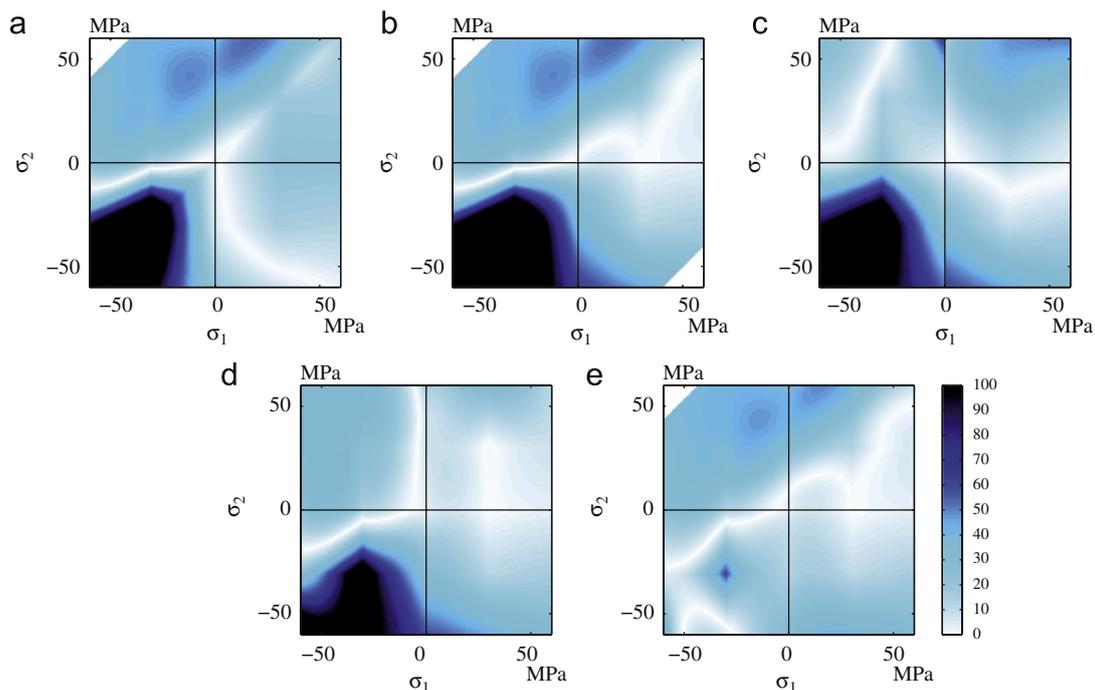


Fig. 21. Error (%) on the magnetic susceptibility estimates in the stress plane for $H=250$ A/m: (a) Kashiwaya (with $K=1$), (b) Schneider and Richardson, (c) Sablik et al., (d) deviatoric and (e) generalized ($k=1.277 \times 10^{-7} \text{ m}^3 \text{ J}^{-1}$) equivalent stresses.

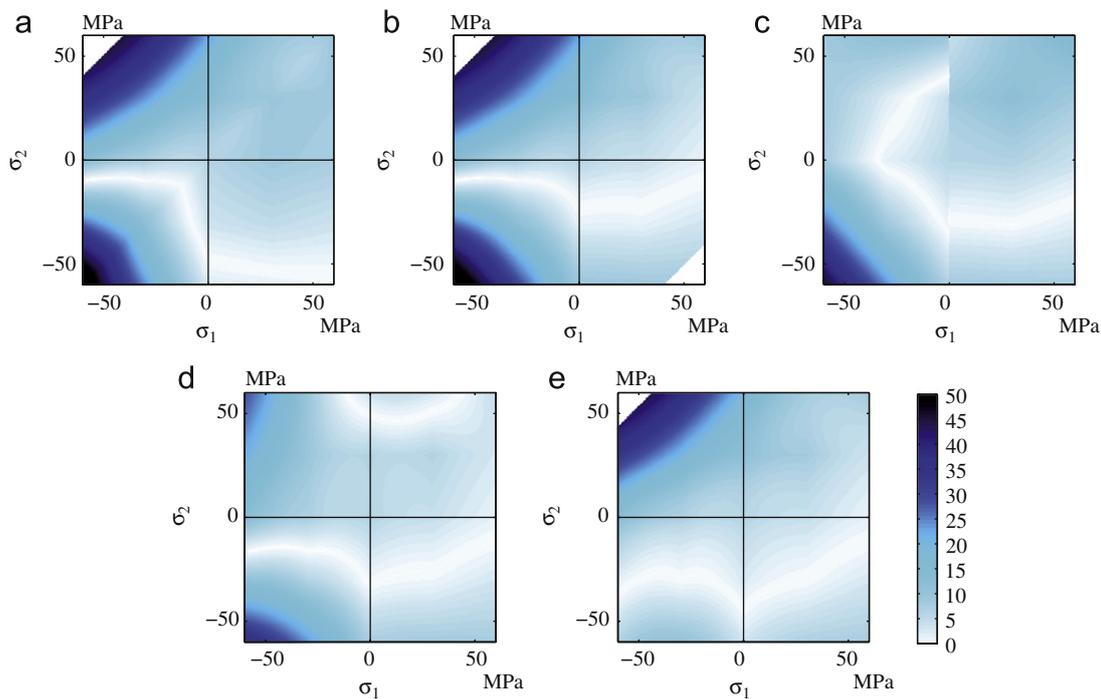


Fig. 22. Error (%) on the magnetic susceptibility estimates in the stress plane for $H=2500$ A/m: (a) Kashiwaya (with $K=1$), (b) Schneider and Richardson, (c) Sablik et al., (d) deviatoric and (e) generalized ($k=1.277 \times 10^{-7} \text{ m}^3 \text{ J}^{-1}$) equivalent stresses.

plotted the evolution of a normalized initial permeability (ratio of initial permeability under stress with initial permeability of the unloaded material) in the biaxial stress plane for a 3%Si-Fe electrical steel. Fig. 23b shows the corresponding results. These results can be compared with the prediction of the generalized equivalent stress. Fig. 23c shows the equivalent stress associated to a parameter $k=1.96 \times 10^{-8} \text{ m}^3 \text{ J}^{-1}$ in adequation

with the two materials investigated (arbitrary units have been chosen). The main difference with the experiments reported in this paper is the low magnitude of λ_m estimated to $\lambda_m=10 \times 10^{-6}$. The variations of the equivalent stress in the sheet plane seem to be in accordance with experimental results. This simulation confirms the ability of prediction of the new generalized equivalent stress.

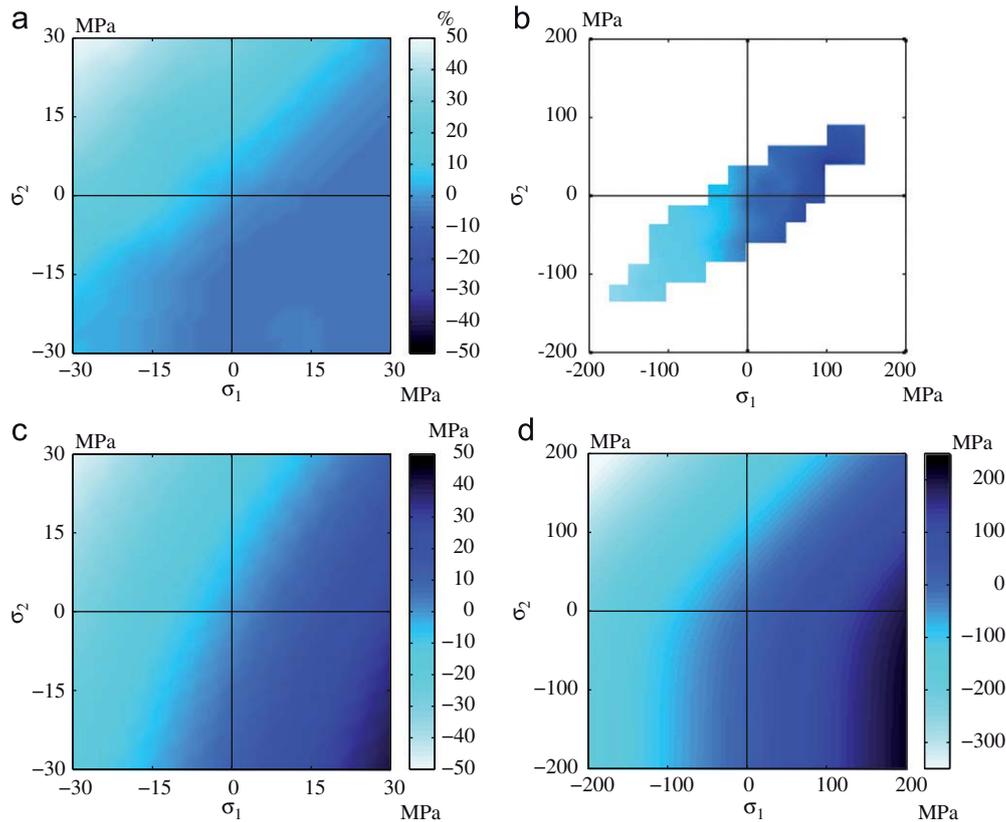


Fig. 23. Prediction of equivalent stress in the stress plane from generalized criterion, applied to iron-based alloys and comparison to previous experimental data – (a) Results from Pearson [27] – coercivity change as a function of biaxial stress. The color scale gives percentage of the stress-free coercivity; (b) results from Maurel [33] – initial normalized permeability – defined as the ratio between permeability under stress and stress-free permeability; (c) equivalent stress map – $k=1.96 \times 10^{-8} \text{ m}^3 \text{ J}^{-1}$ for $\{\sigma_1, \sigma_2\} \in [-30 \text{ MPa}, 30 \text{ MPa}]$ – comparison to results from Pearson; (d) equivalent stress map – $k=1.96 \times 10^{-8} \text{ m}^3 \text{ J}^{-1}$ for $\{\sigma_1, \sigma_2\} \in [-200 \text{ MPa}, 200 \text{ MPa}]$ – comparison to results from Maurel.

Complementary experiments are foreseen in order to validate or extend the equivalent stress proposals:

- experiments involving a magnetic field direction out of the mechanical eigenaxes;
- experiments involving measurement of the hysteresis losses and magnetostriction;

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