Finite Element Modeling of Magnetoelectric Sensors

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The magnetoelectric effect, stemming from piezoelectric and magnetostrictive materials composite, is studied. A model based on the association of magnetoelastic and piezo-electric constitutive laws is presented. This model is implemented in a finite element formulation and a comparison with analytical solutions for piezoelectric/magnetostrictive composite is realized. A magnetoelectric displacement sensor is finally studied.

***Index Terms*—Finite element formulation, magnetoelectric sensors, magnetostriction, piezoelectric.**

I. Introduction

SMART materials such as magnetostrictive (MM) and piezoelectric (PM) materials are usually used in a wide range of electromechanical systems. The strong coupling between electromagnetic and mechanical properties in this type of materials enables to control electric or magnetic (respectively mechanical) behavior by mechanical (respectively electric or magnetic) quantities. These materials are commonly employed separately but they can also be used together in a composite design. The presence of a magnetic field within a MM generates a magnetostriction strain, that transmitted to a PM, is associated to an electric polarization. Conversely, an electric field in a PM can create a modification of magnetization in a MM. This effect is called "magnetoelectric" effect.

Analytical studies of this effect, considering small variations of the fields, with MM/PM laminate composites as well as conception of such structures have been proposed [1][2]. The design of a novel generation of smart systems using this effect needs models describing accurately their behavior associated to robust modeling tools for solving coupled problems, in order to optimize efficiently such structures.

A magnetoelastic model built from a thermodynamical approach is presented. It is based on nonlinear constitutive laws which present the mutual interaction between magnetic and elastic properties. Piezoelectric constitutive laws, in the linear assumptions, are detailed. From the minimization of functional energy, finite element formulation of the magnetoelastic and electroelastic problems are established. Specific considerations allow to establish finite element formulation of magnetoelectric problem. Validation of the formulation is realized with comparison of analytical solutions of two MM/PM composite structures. Finally, study of a magnetoelectric displacement sensor is achieved.

II. Magnetoelectric Modeling

The behavior of active materials, when losses are neglected, is given by the knowledge of the dependence of the electric flux density d and the stress tensor σ on the electric field e and the strain tensor s for PM, and of the magnetic field h and the stress σ on the magnetic flux density b and the strain s for MM

$$\sigma(e,s) \qquad d(e,s) \tag{1}$$

$$\sigma(b,s) \qquad h(b,s). \tag{2}$$

The definition of expressions (1) and (2) requires the use of piezoelectric coefficients α [3] as well as piezomagnetic coefficients γ [4]

$$\alpha_{ikl} = \frac{\partial d_i}{\partial s_{kl}} = -\frac{\partial \sigma_{kl}}{\partial e_i} \tag{3}$$

$$\gamma_{ikl} = \frac{\partial h_i}{\partial s_{kl}} = \frac{\partial \sigma_{kl}}{\partial b_i}. \tag{4}$$

A. Electroelastic Behavior

PM are usually used around a polarization point. In this case, all the material parameters are constant and the behavior is taken linear. A simple integration of the piezoelectric coefficients (3) gives the following expression [3] of the electroelastic behavior:

$$\begin{aligned} \sigma_{ij}(e,s) &= C^e_{ijkl}\, s_{kl} - \alpha^t_{kij}\, e_k \\ d_i(e,s) &= \alpha_{ikl}\, s_{kl} + \varepsilon^s_{ij}\, e_j \end{aligned} \tag{5}$$

where C^e and ε^s are respectively the stiffness tensor at constant electric field and the electrical permittivity at constant strain. In linear piezoelectricity, the equations of linear elasticity are coupled to the charge equation of electrostatics by the mean of the piezoelectric coefficients.

B. Magnetoelastic Behavior

The magnetostrictive behavior is highly non linear, and this non linearity has to be considered in the magnetoelastic constitutive laws. The mechanical behavior law is written in the framework of linear elasticity, using the decomposition of total strain into elastic strain s^e and magnetostriction strain s^μ, $s_{kl} = s^e_{kl} + s^\mu_{kl}$ [5]. Besides, magnetostriction strain induced by a magnetic field is assumed to depend only on the magnetic flux density. With these assumptions Hooke's law is expressed as follows:

$$\sigma_{ij}(b,s) = C_{ijkl}(s_{kl} - s^\mu_{kl}(b)) \tag{6}$$

with s_{kl} the total strain tensor and C_{ijkl} the usual stiffness tensor defined, in the case of isotropic material, by

$$C_{ijkl} = \frac{E^*}{1+\nu^*}\left(\frac{\nu^*}{1-2\nu^*}\delta_{kl}\delta_{ij} + \frac{1}{2}(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk})\right) \tag{7}$$

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where E^*, ν^* and δ are respectively Young's modulus, Poisson's ratio and the Kronecker's symbol.

From the definition of the piezomagnetic coefficients (4), integration of both terms between s^μ and s enables to express the magnetic behavior law $h(b,s)$. This law can be written by introducing a coercive magnetic field h^c which describes the effect of an applied stress

$$\begin{aligned} h_i(b,s) &= h_i^0(b,s^\mu) - C_{klnp}\frac{\partial s_{np}^\mu(b)}{\partial b_i}(s_{kl} - s_{kl}^\mu) \\ &= h_i^0(b,s^\mu) - h_i^c(b,s) \end{aligned} \tag{8}$$

$h_i^c(b,s)$ is the magnetic field induced along i axis by stress at given magnetic flux density and $h_i^0(b,s^\mu)$ is the magnetic field at free stress depending only on magnetic flux density.

C. Magnetostriction Strain Model

The magnetostriction strain tensor $(s_{//}^\mu, s_{\perp_1}^\mu, s_{\perp_2}^\mu)$ is expressed in the reference frame of the magnetic induction. The component $s_{//}^\mu$ can be approximated as a polynomial function versus the magnetic flux density [6]. Besides, assuming magnetostriction phenomena isochore and isotropic, the magnetostriction strain tensor can be expressed in the reference frame of the magnetic induction $(b_{//}, b_{\perp_1}, b_{\perp_2})$:

$$s_{//}^\mu = \sum_{n=0}^{N} \beta_n b^{2(n+1)} \quad s_{\perp_1}^\mu(b) = s_{\perp_2}^\mu(b) = -\frac{s_{//}^\mu}{2}. \tag{9}$$

To take into account the magnetic flux density distribution, the magnetostriction strain tensor in the material frame is given by the following indicial form:

$$s_{kl}^\mu(b) = \frac{1}{2}\sum_{n=0}^{N} \beta_n\, b^{2n}(3b_k b_l - \delta_{kl} b^2). \tag{10}$$

From the expression of the magnetostriction strain tensor, the coercive magnetic field (8) can be expressed as the product of an 'equivalent' reluctivity tensor with the magnetic flux density

$$h_i^c(b,s) = \nu_{ij}^c(b,s) b_j. \tag{11}$$

Due to the applied stress, the 'equivalent' reluctivity tensor is anisotropic. Its expression can be found in [6]. A reasonable first approximation for the magnetostriction strain tensor can be obtained by neglecting the terms up to $N = 1$.

III. Finite Element Formulation

A. Magnetoelastic Problem

In the static case, the finite element formulation integrating the magnetostrictive phenomena can be established from a minimization of the functional energy E in terms of b and s:

$$E(b,s) = W(b,s) - T \tag{12}$$

where $W(b,s)$ and T are respectively the magnetoelastic energy and the work of magnetic and mechanical sources, defined by

$$W(b,s) = \int_{\Omega_T}\left(\int_0^b h^0(b', s^\mu)db' + \int_{s^\mu}^{s} \sigma(b,s')ds'\right)d\Omega_T \tag{13}$$

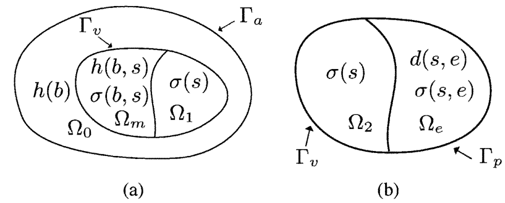

Fig. 1. The studied domains.

$$\begin{aligned} T = &\int_{\Omega_T} a\cdot j\, d\Omega + \int_{\Gamma_h} a\cdot(h\times n)\, d\Gamma_h \\ &+ \int_{\Omega_M} u\cdot f^\Omega\, d\Omega + \int_{\Gamma_\sigma} u\cdot(\sigma\cdot n)\, d\Gamma_\sigma \end{aligned} \tag{14}$$

with a the magnetic vector potential, u the vector displacement, j the current density, f^Ω the volume force density and n the normal vector. Boundary conditions associated to the magnetomechanical problem are of two types

$$\begin{aligned} h\times n = 0 \quad &\text{on} \quad \Gamma_h \qquad \sigma\cdot n = f^\Gamma \quad \text{on} \quad \Gamma_\sigma \\ b\cdot n = 0 \quad &\text{on} \quad \Gamma_b \qquad\quad u = 0 \quad \text{on} \quad \Gamma_u \end{aligned}$$

where $\Gamma_a = \Gamma_h \cup \Gamma_b$ and $\Gamma_v = \Gamma_\sigma \cup \Gamma_u$ are the boundaries of the study domains defined by $\Omega_T = \Omega_M \cup \Omega_0$ and $\Omega_M = \Omega_m \cup \Omega_1$ [Fig. 1(a)]. f^Γ is associated to surface force densities.

Application of variational principles, gives the following magnetic and mechanical formulation associated to arbitrary variations δ_a and δ_u:

$$\begin{aligned} &\int_{\Omega_T} \nabla\times\delta_a[\nu]\nabla\times a d\Omega_T + \int_{\Gamma_h}\delta_a\cdot(h\times n)d\Gamma_h \\ &\quad = \int_{\Omega_m}\nabla\times\delta_a\cdot h^c(b,s)\, d\Omega_T + \int_{\Omega_T}\delta_a\cdot j\, d\Omega_T \end{aligned} \tag{15}$$

$$\begin{aligned} &\int_{\Omega_1} s(\delta_u)[C]s\, d\Omega + \partial_u\left(\frac{1}{2}\int_{\Omega_T}\left(\int_0^b h^0(b', s^\mu)db'\right)d\Omega_T\right) \\ &\quad = \int_{\Omega_m} s(\delta_u)[C]s^\mu d\Omega + \int_{\Omega_m}\delta_u\cdot f^\Omega\, d\Omega + \int_{\Gamma_\sigma}\delta_u\cdot f^\Gamma d\Gamma_\sigma. \end{aligned} \tag{16}$$

The magnetic vector potential is discretized using edge and nodal elements, respectively in 3D and 2D cases, while the displacement vector is discretized with nodal elements. From this discretization, the algebraic equation system is defined by

$$\begin{aligned} [M](a) &= (J) + (J^c(b,s)) \\ [K](u) &= (F^\sigma) + (F^{mf}(b)) + (F^\mu(b)). \end{aligned} \tag{17}$$

The right-hand side of each equation (17) can be interpreted as a coercive current density J^C representing the effect of an applied stress and the 'equivalent' nodal magnetostriction forces F^μ resulting from an applied magnetic field. F^{mf} are the nodal magnetic forces determined by the local derivative of magnetic energy [7]. The obtained algebraic system is nonlinear. It can be solved by an iterative procedure based on fixed point algorithm, where the nonlinearity due to magnetic properties and magnetostriction effects is placed at the right-hand side of each equation. In this case, the magnetic $[M]$ and mechanical $[K]$ rigidity matrices are calculated only at the first iteration.

B. Electroelastic Problem

Electromechanical problem in the static condition can be defined, in a similar way, by minimization of a functional energy. This one, integrating piezoelectric phenomena, leads to solve the following equality of the electrical and mechanical problems associated to the variations $\delta\varphi$ and δu [8] [see (18) and (19)]

$$\int_{\Omega_e} \nabla\delta\varphi\,[\alpha]\,s\,d\Omega_p - \int_{\Omega_e} \nabla\delta\varphi\,[\varepsilon]\,\nabla\varphi\,d\Omega_p = \int_{\Gamma_d} \delta\varphi\,(d\cdot n)\,d\Gamma_p \quad (18)$$

$$\int_{\Omega_2} s(\delta u)\,[C]\,s\,d\Omega_p + \int_{\Omega_e} s(\delta u)\,[\alpha]^t\,\nabla\varphi\,d\Omega_p - \int_{\Omega_p} \cdot\delta u\cdot f^{\Omega}\,d\Omega_p - \int_{\Gamma_\sigma} \delta u\cdot f^{\Gamma}\,d\Gamma_\sigma = 0. \quad (19)$$

Boundary conditions relative to the electroelastic problem, where $\Omega_p = \Omega_e \cup \Omega_2$ and $\Gamma_p = \Gamma_d \cup \Gamma_e$ [Fig. 1(b)], are defined by

$$e\times n = 0 \quad \text{on} \quad \Gamma_e \qquad d\cdot n = 0 \ \text{on} \ \Gamma_d$$
$$\sigma\cdot n = f^{\Gamma} \ \text{on} \ \Gamma_\sigma \qquad u = 0 \ \text{on} \ \Gamma_u.$$

After discretisation with nodal elements, the algebraic equation system relative to the electroelastic problem is

$$\begin{bmatrix} [K_{uu}] & [K_{u\phi}] \\ -[K_{u\phi}]^t & [K_{\phi\phi}] \end{bmatrix} \begin{pmatrix} u \\ \phi \end{pmatrix} = \begin{pmatrix} F \\ Q \end{pmatrix} \quad (20)$$

with ϕ the electric potential and Q the vector of the nodal electric charges. F can be taken equal to the external forces F^{σ}. For piezoelectric material the electrostatic forces, due to the form effect, can be neglected.

C. Magnetoelectric Problem

The sensor configuration of the piezoelectric materials is a particular case. In this situation, the total electrical charge Q can be considered equal to zero. The second equation of (20) is simplified and it is possible to write

$$(\phi) = [K_{\phi\phi}]^{-1}[K_{u\phi}]^t(u) \quad (21)$$
$$[K] = [K_{uu}] + [K_{u\phi}][K_{\phi\phi}]^{-1}[K_{u\phi}]^t. \quad (22)$$

Equation (22) gives, for the piezoelectric material, the effect of an equivalent piezoelectric stiffness induced by electric potentials on the sensor electrodes. It allows to consider the influence of the inverse piezoelectric effect in the global structure rigidity matrix during the mechanical resolution of a magnetomechanical problem. The rigidity matrix associated to piezoelectric material has to be modified considering (22) in the mechanical equation of the magnetomechanical problem (17). Consequently, magnetoelectromechanical problems are coupled into a single magnetomechanical code. Electrostatic potential is obtained after convergence of the magnetomechanical problem by use of (21).

IV. Model Validation

In order to validate the formulation, let us first consider two composite structures. The first one is a MM/PM bilayer [Fig. 2(a)], respectively of FeCo and PZT materials, while the second is a multilayer with one MM and two PM with the same materials [Fig. 2(b)].

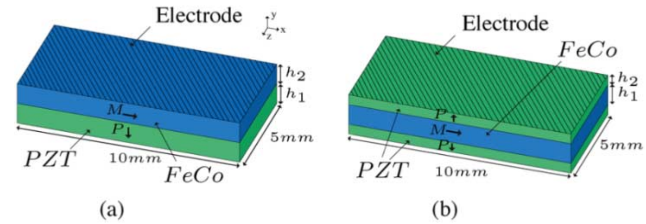

Fig. 2. Two configurations of MM/PM composite.

A. Analytical Solution

To obtain an analytical solution, some considerations are required. First, one dimension (along x axis) is significantly longer than the others, second, *Kirchhoff* assumption is used. Moreover, the volume of PM and MM are assumed identical (see Fig. 2). This gives the following composite strain expression versus y distance:

$$s(y) = s_0 - \frac{y}{r} \quad (23)$$

where s_0 is the mid plane strain and r the composite curvature. From the mechanical balance equations, forces and moments, equations closing PM strain to MM strain relative to the multilayer (24) and the bilayer (25) are defined by

$$\frac{s^{pzt}}{s^{\mu}} = \frac{E_1}{E_1 + E_2} \quad (24)$$
$$\frac{s^{pzt}}{s^{\mu}} = \frac{E_1(E_1 + E_2)}{E_1^2 + E_2^2 + 14E_1E_2} \quad (25)$$

where E_1 and E_2 are respectively the Young's modulus of the PM and MM. From the knowledge of strains, electric potential is deduced. To obtain these results, the inverse piezoelectric effect is neglected.

B. Comparison of Analytical and Numerical Solutions

PM and MM are made respectively of EB10 ceramic and FeCo ferromagnetic material. It appears that the electric potential (Fig. 3) is higher for the multilayer than for the bilayer, considering the same quantities of piezoelectric material in the two structures. Besides, it can be shown that the ratio between electric potential of the bilayer (V_b) and the multilayer (V_t), for the analytical and numerical solutions, are respectively of $V_b/V_t = 0.295$ and $V_b/V_t = 0.3$. Analytical and numerical solutions are in good agreement.

V. Magnetoelectric Displacements Sensor

Taking the MM/MP multilayer as a basis, design of novel smart material structures is possible. One of these, which will be taken as an example, is a magnetoelectric displacements sensor [9], intended to detect the displacement of ferromagnetic objects. This sensor is constituted of a laminate composite associated to two iron yokes and a permanent magnet (Fig. 4). The laminate composite is a MM plate bonded with two PM plates. Magnetic orientation and electric polarization are orthogonal, as indicated in Fig. 4. Both sides of the composite are stuck (not

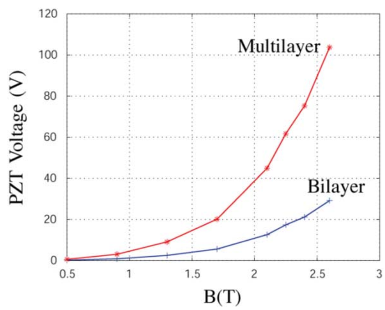

Fig. 3. Electric potential versus magnetic flux density.

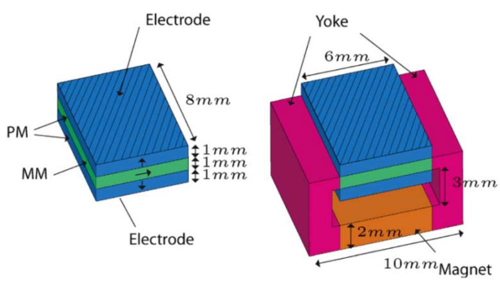

Fig. 4. Magnetoelectric displacements sensors.

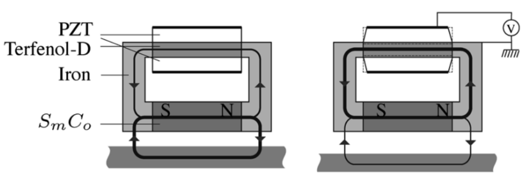

Fig. 5. Magnetic flux distribution (left) and its change with ferromagnetic plate displacement (right).

bonded) to the iron yokes with magnetic forces via flux path. In this structure, small part of the magnetic flux is derived between the mobile ferromagnetic plate and the MM (Terfenol-D). Thereafter, small variations of the air gap change the magnetic flux inside the MM, which is associated to magnetostriction strain variation. This strain is then responsible for an electric field. Thus, the magnetoelectric device converts the displacement into a voltage on the electrodes (Fig. 5). The multilayer composite has advantage to reduce bending deformation due to magnetostriction.

We intend to study the sensitivity of the sensor linked to the air gap variations. For that, in a first time MM is considered at free stress. Due to the magnetic flux distribution, electric potential are not equal into the two PM [Fig. 6(a)]. A small bending effect is present. In comparison with the experimental results, sensitivity of the lower PM is in good adequacy [Fig. 6(b)]. Differences are observed for the small air gaps. If the Terfenol-D is pre-stressed, magnetostriction strain developed is more important [6]. If we consider that, with an external set, the Terfenol-D layer of the sensor can be pre-stressed, it results in an increase of the electric potential [Fig. 6(a)] and a higher sensitivity of the sensor [Fig. 6(b)].

This analysis allows a good description of the operating mode of the sensor, and the developed model can be useful for design and optimization of new structures.

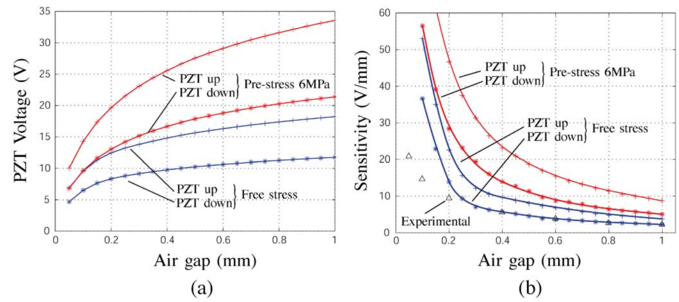

Fig. 6. Response of the magnetoelectric displacements sensor.

VI. Conclusion

A finite element model has been developed in order to study the magnetoelectric effect, stemming from magnetostrictive and piezoelectric materials composite. Standard piezoelectric behavior laws are considered, while the coupling of magnetic and elastic properties is ensured by nonlinear constitutive laws. Specific considerations allow a coupling between the magnetoelastic and the electroelastic problems, such that only the resolution of a magnetoelastic problem is necessary. To validate the approach, a comparison between analytical and numerical solutions of multilayers is globally satisfactory. Finally this model is successfully used to analyze a new device of magnetoelectric displacement sensor, based on the variations of the electric potential due to air gap changes. Some discrepancies are observed with experimental results. They may be due to the imperfect definition of mechanical boundary conditions. The properties of the permanent magnet may also not be identical to the experimental device and the influence of the PM sticking with the MM has not been taken into account.

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