Equivalent Stress Criteria for the Effect of Stress on Magnetic Behavior

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A main limitation of most models describing the effect of stress on the magnetic behavior is that they are restricted to uniaxial, tensile or compressive, stress. An idea to overcome this strong limitation is to define a fictive uniaxial stress, the equivalent stress that would affect the magnetic behavior in a similar manner than a multiaxial one. Several authors have tried to define such a criterion. We propose in this paper to compare several equivalent stress definitions, and to apply them in the case of uniaxial and biaxial mechanical loadings for which experimental results are available.

***Index Terms*—Effect of stress, equivalent stress, magneto-elasticity, multiaxiality.**

I. Introduction

In most practical electromagnetic applications, magnetic materials are submitted to multiaxial stress inherited from forming process or appearing in use. These stress states can change significantly the magnetic behavior of materials [1]. However, the few available models describing the effect of stress on the magnetic behavior are usually restricted to uniaxial (tensile or compressive) stress (see for instance [2]–[4]). A solution to introduce the multiaxiality of stress into modeling tools is the definition of an equivalent stress criterion. An equivalent stress for the magnetic behavior is a (fictive) uniaxial stress that would change the magnetic behavior in a similar manner than the multiaxial one[1] . This approach follows the classical equivalent stress definitions used in mechanics such as Von Mises or Tresca equivalent stresses for plasticity. Several equivalent stress for magneto-elasticity have been proposed in the past years [5]–[9]. We propose in this paper to compare these proposals. Experimental results carried out under biaxial mechanical loading will allow to validate these criteria.

II. Several Equivalent Stress Definitions

Several authors tried to define an equivalent stress for magneto-elastic behavior, usually thanks to energetic considerations and experimental observations of magnetic behavior of materials submitted to biaxial mechanical loadings.

Kashiwaya (K) [5] proposed the following definition for the equivalent stress $\sigma_{\mathrm{eq}}^{\mathrm{K}}$:

$$\sigma_{\mathrm{eq}}^{\mathrm{K}} = K(\sigma_1 - \sigma_{\max}) \tag{1}$$

with K a constant, σ_1 the eigenstress aligned with the magnetic field direction and $\sigma_{\max}$ the maximal value of the stress tensor eigenvalues. This equivalent stress is always negative or null. Iso-values are parallel lines in the (σ_1, σ_2) plane. If the magnetic field is applied along the direction of the maximum eigenstress, the equivalent stress is zero, so that a tensile stress or an equibiaxial tension or compression are supposed to have no effect on the magnetic behavior.

Schneider and Richardson (SR) [6] proposed the following definition for the equivalent stress $\sigma_{\mathrm{eq}}^{\mathrm{SR}}$:

$$\sigma_{\mathrm{eq}}^{\mathrm{SR}} = \sigma_1 - \sigma_2 \tag{2}$$

σ_1 and σ_2 are the eigenstresses in the sheet plane, the magnetic loading being aligned in the direction of σ_1. The main difference with K definition is that the area of the stress plane where $\sigma_1 > 0$ and $\sigma_2 < 0$ defines a positive equivalent stress. But an equibiaxial stress is still supposed to have no effect on the magnetic behavior.

Sablik and co-workers (S) [7] proposed the following definition for the equivalent stress $\sigma_{\mathrm{eq}}^{\mathrm{S}}$, based on previous magneto-mechanical measurements by Langman [10]:

$$\begin{cases} \sigma_{\mathrm{eq}}^{\mathrm{S}} = \frac{1}{3}(2\sigma_1 - \sigma_2) & \text{for } \sigma_1 < 0 \\ \sigma_{\mathrm{eq}}^{\mathrm{S}} = \frac{1}{3}(\sigma_1 - 2\sigma_2) & \text{for } \sigma_1 \geq 0 \end{cases} \tag{3}$$

σ_1 is still the stress aligned with the magnetic field. Equi-bitraction and equi-bicompression do not lead to the same result, that is a significant difference with K and SR approaches. But S model is discontinuous for $\sigma_1 = 0$.

Pearson and co-workers [8] also proposed an equivalent stress for a biaxial mechanical loading. In its simplest form this equivalent stress corresponds to SR proposal. The more refined form is a polynomial interpolation that reveals complicated to use because the parameter identification is sample dependent.

Daniel and Hubert (DH) [9] proposed the following definition $\sigma_{\mathrm{eq}}^{\mathrm{DH}}$, based on an equivalence in magneto-elastic energy:

$$\sigma_{\mathrm{eq}}^{\mathrm{DH}} = \frac{3}{2} {}^t\vec{h}\boldsymbol{s}\vec{h} \tag{4}$$

$\vec{h}$ is the direction parallel to the applied magnetic field and $\boldsymbol{s}$ is the deviatoric part of the stress tensor $\boldsymbol{\sigma}(\boldsymbol{s} = \boldsymbol{\sigma} - 1/3\,\mathrm{tr}(\boldsymbol{\sigma})\boldsymbol{I})$. It can be noticed that the equivalent stress is zero when the stress is hydrostatic, meaning that a hydrostatic pressure has no effect on the magnetic behavior. A main advantage of this criterion is that it can be applied to a fully multiaxial mechanical loading, whereas the previous proposals only refer to biaxial stress state.

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[1]Meaning that an uniaxial stress with the amplitude defined by the equivalent stress criterion should lead to the same shift in magnetic susceptibility than the real multiaxial stress.

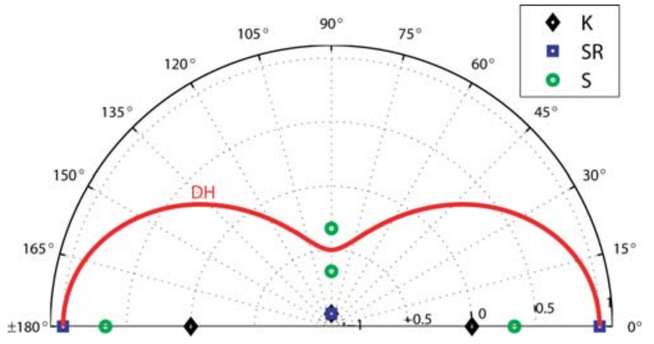

Fig. 1. Equivalent stress $\sigma_{\rm eq}/\sigma_o$ in the case of a uniaxial stress σ_o applied in a direction $\theta \in [0, \pi]$ with respect to the magnetic field (direction 0°).

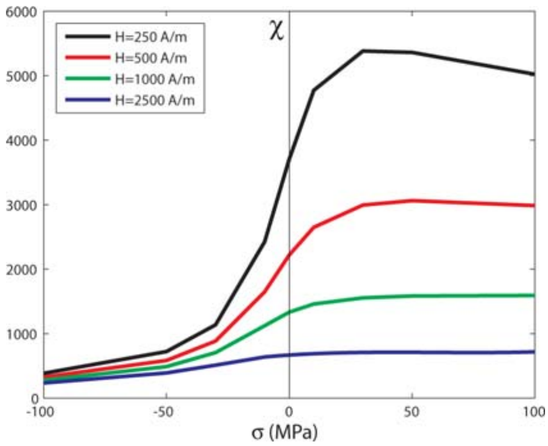

Fig. 2. Experimental secant susceptibility under mechanical loadings.

III. Application in the Case of Uniaxial Tension or Compression

As a first analysis, these equivalent stress criteria can be compared to each other in the case of a uniaxial mechanical loading (intensity σ_0). The result is presented in Fig. 1.

K criterion can only be applied if the maximum eigenstress is in the direction of the magnetic field. SR and S criteria can only be applied in the case when the direction of the magnetic field is a principal direction for the stress. DH criterion can be applied whatever the relative orientation between stress and magnetic field. In the uniaxial case, $\sigma_{\rm eq}^{\rm DH}$ can be written:

$$\sigma_{\rm eq}^{\rm DH}(\theta) = \frac{3}{2}\sigma_o\left(\cos^2\theta - \frac{1}{3}\right). \tag{5}$$

It has to be noticed that in the case of an uniaxial stress applied in the direction of the magnetic field, K and S proposals differ from the applied stress. For SR and DH proposals, the equivalent stress reduces, as expected, to the applied stress. The discontinuity of S model for $\theta = \pi/2(\sigma_1 = 0)$ is also highlighted.

Some measurements of the susceptibility of an iron-cobalt under uniaxial stress loading have been carried out using a uniaxial magneto-mechanical set-up. The corresponding results are presented in Fig. 2 for several magnetic field levels. The susceptibility change is higher in compression $\sigma < 0$ than in tension $\sigma < 0$.

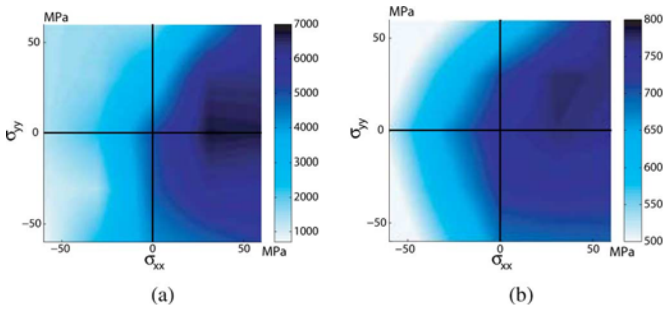

Fig. 3. Experimental secant susceptibility under mechanical loadings. (a) $H = 250\ \mathrm{A{\cdot}m^{-1}}$, (b) $H = 2500\ \mathrm{A{\cdot}m^{-1}}$.

IV. Application in the Case of Biaxial Mechanical Loadings

Experiments have been performed on iron-cobalt laminations (cross specimen) [11]. They consist in anhysteretic magnetic measurements carried out under biaxial mechanical stress in homogeneous magnetic and mechanical conditions, for stress levels varying from -60 MPa to $+60$ MPa. Magnetic field is applied along direction 1. The results are shown in Fig. 3.

Tension $(\sigma_1 > 0, \sigma_2 = 0)$ slightly increases the susceptibility, and compression $(\sigma_1 < 0, \sigma_2 = 0)$ strongly decreases it. An uniaxial stress in the direction orthogonal to the field $(\sigma_1 = 0, \sigma_2 \neq 0)$ deteriorates the magnetic behavior, with a stronger effect in tension. Equibiaxial stress $(\sigma_1 = \sigma_2)$, and shear stress $(\sigma_1 = -\sigma_2)$ strongly deteriorate the magnetic behavior when σ_1 is negative, and have a much lower effect when σ_1 is positive.

The expected susceptibility according to K, SR, S and DH criteria has been estimated (the experimental data for the susceptibility under uniaxial mechanical loading have been extracted from the measurements with $\sigma_2 = 0$) [2] . The experimental conditions correspond to biaxial stress (σ_1, σ_2) with the magnetic field applied along eigendirection 1 (DH equivalent stress is then defined by $\sigma_{\rm eq} = \sigma_1 - (1/2)\sigma_2$). Fig. 4 shows a map of the relative error between predicted χ_p and measured χ_e susceptibilities $(e = 100 \times |\chi_p - \chi_e|/\chi_e)$ for a magnetic field of 250 $\mathrm{A{\cdot}m^{-1}}$.

For all criteria, the errors observed in equibicompression are very high. In the case of a stress $(\sigma_1, \sigma_2) = (-60\ \mathrm{MPa}, -60\ \mathrm{MPa})$ the error is up to 635% for K and SR, 371% for S and 238% for DH. All the proposed equivalent criteria fail in the prediction of the effect of a bicompressive stress. If the error values are truncated to 50% for reading convenience, Fig. 5 is obtained.

It appears that outside the bicompression area, DH criterion is closest to experimental results. Fig. 6 has been plotted for a magnetic field of 2500 $\mathrm{A.m^{-1}}$. The same comments can be made. Errors are lower in that case. This decrease of the error is linked to the fact that for such a level of magnetic field, close to saturation, the effect of stress on the magnetic behavior is less sensitive (it can be observed from Figs. 2 and 3).

[2]Complementary measurements of Fig. 2 have been made mainly to ensure that the 1-D measurements with the cross specimen were consistent with a real 1-D configuration.

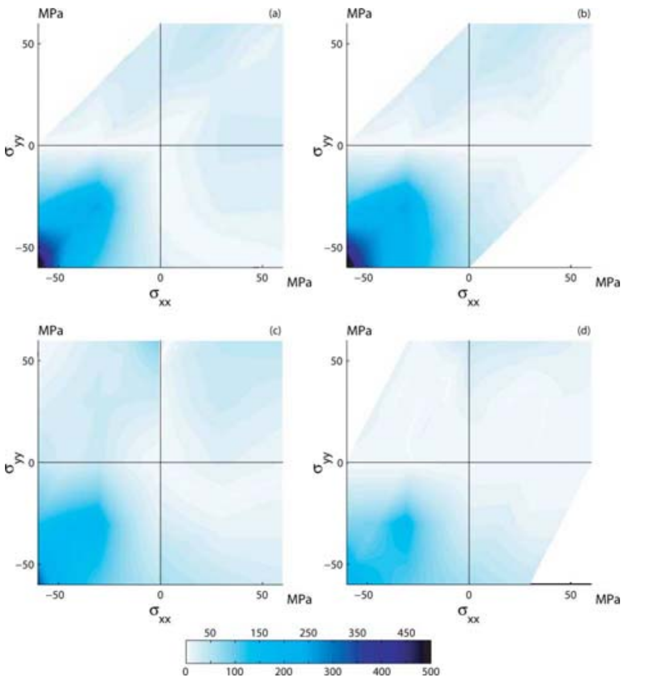

Fig. 4. Relative error (percent) for the predicted susceptibility ($H = 250$ A/m) according to several equivalent stress proposals. (a) K ($K = 1$), (b) SR, (c) S, (d) DH.

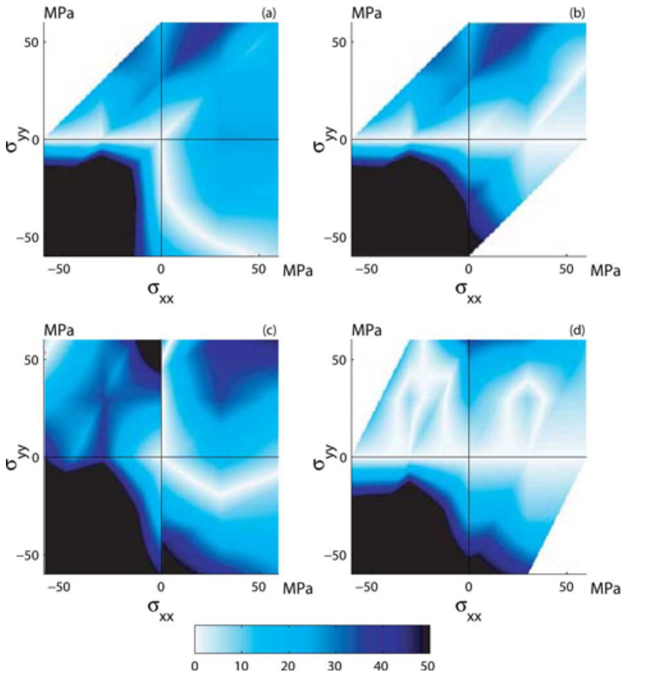

Fig. 5. Relative error (percent) for the predicted susceptibility ($H = 250$ A/m) according to several equivalent stress proposals. (a) K ($K = 1$), (b) SR, (c) S, (d) DH.

V. Guidelines for the Practical Implementation of an Equivalent Stress

The use of an equivalent stress criterion for magneto-elastic behavior is required when an electromagnetic device under significant mechanical loading is studied. As an example, we can consider the stress state inherited from a binding process. For the sake of simplicity, we will consider the very simple configuration of a rigid cylindrical yoke of internal diameter ϕ_0 binding a cylinder of external diameter $\phi_e = \phi_0 + 2\Delta$ and internal diameter ϕ_i. Under plain strain assumptions, the corresponding stress tensor in the cylinder as a function of the radius r is given in cylindrical coordinates by (6)

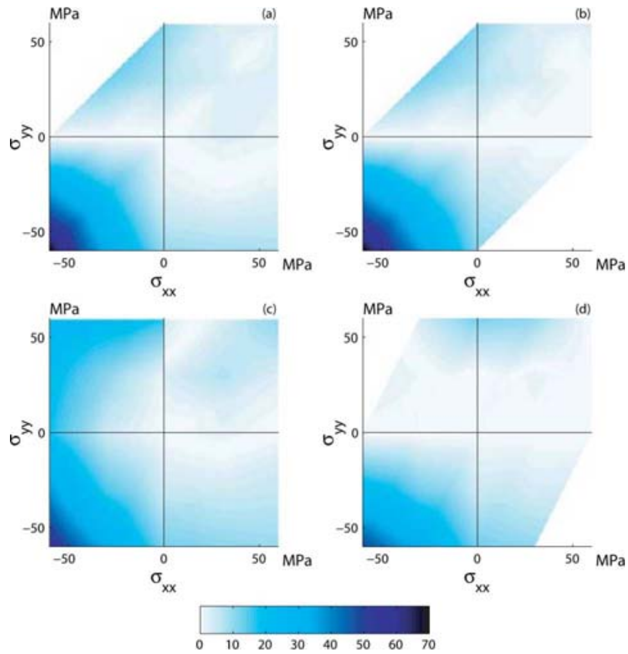

Fig. 6. Relative error (percent) for the predicted susceptibility ($H = 2500$ A/m) according to several equivalent stress proposals. (a) K ($K = 1$), (b) SR, (c) S, (d) DH.

$$\boldsymbol{\sigma}(r) = \begin{pmatrix} \sigma_{rr} & 0 & 0 \\ 0 & \sigma_{\theta\theta} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}_{r\theta z} \tag{6}$$

with

$$\begin{cases} \sigma_{rr} = -K\Delta \left(1 - \left(\dfrac{\phi_i}{2r}\right)^2\right), \\ \sigma_{\theta\theta} = -K\Delta \left(1 + \left(\dfrac{\phi_i}{2r}\right)^2\right), \\ \sigma_{zz} = -K\Delta \, \dfrac{\lambda}{\mu + \lambda}, \end{cases} \tag{7}$$

and

$$K = \frac{4\mu(\mu + \lambda)\phi_e}{\mu\phi_e^2 + (\mu + \lambda)\phi_i^2}. \tag{8}$$

λ and μ are the Lamé coefficients defining the elastic properties of the material (under isotropic assumption). Introducing the effect of such a triaxial stress state on the magnetic behavior would require to have an access to the characterization of the magnetic behavior under complex multiaxial mechanical loading. These experimental data are usually not available. The proposed alternative is to compute the equivalent uniaxial stress given by (4). From (6), and considering a magnetic field in the direction $\vec{h} =^t [h_r \;\; h_\theta \;\; h_z]$ we obtain:

$$\sigma_{\text{eq}}^{\text{DH}} = \frac{3}{2}\left(h_r^2\,\sigma_{rr} + h_\theta^2\,\sigma_{\theta\theta} + h_z^2\,\sigma_{zz}\right) - \frac{1}{2}\left(\sigma_{rr} + \sigma_{\theta\theta} + \sigma_{zz}\right). \tag{9}$$

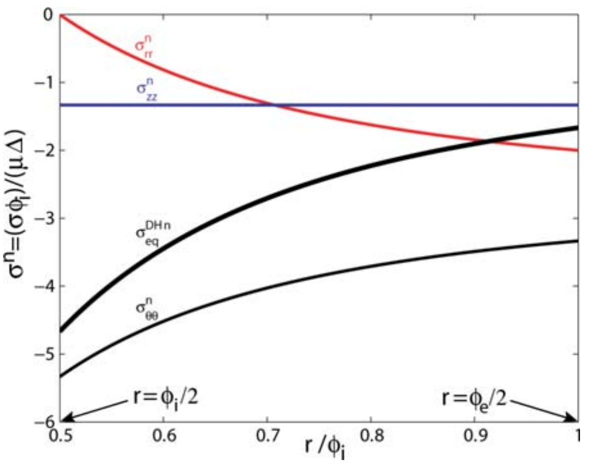

Fig. 7. Normalized values of $\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}$, and $\sigma_{\rm eq}^{\rm DH}$ for a simplified binding configuration with orthoradial magnetic field ($\phi_e/\phi_i = 2$ and $\nu = 0.25$).

It is then assumed that an uniaxial stress parallel to the local magnetic field, with amplitude $\sigma_{\rm eq}^{\rm DH}$, modifies the permeability as the real multiaxial stress. The only necessary data are the characterization of the magnetic behavior under uniaxial stress, that are rather classical measurements.

If we consider an even more simplified configuration where the field is orthoradial $\vec{h} =^t [0\ 1\ 0]$ (e.g., if a current along $\vec{z}$ is imposed in the central wire of diameter ϕ_i), the equivalent stress, applied in the orthoradial direction, reduces to (10)

$$\begin{aligned}\sigma_{\rm eq}^{\rm DH} &= \sigma_{\theta\theta} - \frac{1}{2}(\sigma_{rr}+\sigma_{zz})\\ &= -\frac{K\Delta}{2}\left(\frac{\mu}{\mu+\lambda}+3\left(\frac{\phi_i}{2r}\right)^2\right). \end{aligned} \tag{10}$$

This case is illustrated in Fig. 7, considering $\phi_e = 2\phi_i$ and a Poisson's ratio $\nu = 0.25$. It gives $\lambda = \mu$, $K = (8\mu)/(3\phi_i)$ and we obtain $\sigma_{\rm eq}^{\rm DH} = (-2\mu\Delta)/(3\phi_i)(1+6(\phi_i/2r)^2)$. It can be noticed that the equivalent stress is different from all the principal components of the stress tensor.

In the case of the structural analysis of a more complicated structure under more complicated loadings, the same scheme can be applied. The distribution of stress is first computed, so that the corresponding distribution of the equivalent stress can be calculated. The magnetic computation is then made using magnetization curves under uniaxial stress. The approach proposed in [12] can be used, the value for the uniaxial stress must just be replaced by the equivalent stress.

VI. Conclusion

Several equivalent stress criteria for the effect of multiaxial stress on the magnetic behavior have been compared. The main weakness of these models is their inability to describe the effect of a bicompression stress on the magnetic behavior. Only one equivalent stress (DH) can describe fully multiaxial stress state, without any hypothesis concerning the relative orientation of the magnetic field direction in the principal stress coordinate system. This latter proposal is also the closest to experimental results obtained in biaxial configurations. The guidelines for the use of such equivalent stress criteria have been given in the case of coupled magneto-mechanical structural analysis. These equivalent stress criteria are a strong approximation but they provide a significant—and easy to implement—improvement to the classical uniaxial approach of magneto-elastic couplings. If the corresponding predictions are not accurate enough—for example under bicompressive mechanical loadings—fully coupled multiaxial constitutive laws are required.

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