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On some approaches to model reversible magnetization processes

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Abstract

This paper focuses on the problem of how reversible magnetization processes are taken into account in contemporary descriptions of hysteresis curves. For comparison, three versions of the phenomenological $T(x)$ model based on hyperbolic tangent mapping are considered. Two of them are based on summing the output of the hysteresis operator with a linear or nonlinear mapping. The third description is inspired by the concept of the product Preisach model. Total susceptibility is modulated with a magnetization-dependent function. The models are verified using measurement data for grain-oriented electrical steel. The proposed third description represents minor loops most accurately.

Keywords: hysteresis, modeling, reversibility

(Some figures may appear in colour only in the online journal)

1. Introduction

An important aspect of hysteresis modelling is the proper description of reversible magnetization processes. These may affect the results of calculations for transients, leading to possible misinterpretations of first order reversal curves (FORCs), readily used by solid state physicists in the analysis of magnetic recording media, paleomagnetic samples, nano-arrays and nano-wires [1–8].

This effect may result in potential problems for practical applications like non-destructive testing (NDT) methods [9–12] or prediction of inrush currents in electrical power engineering systems [13–15]. The crucial role of irreversible and reversible processes is particularly visible in the analysis of coupled problems e.g. the magneto-mechanical effect [16–26]. Information on the ratio of reversible versus irreversible magnetization processes at a given induction level allows one to tailor up the properties of soft magnetic materials [27, 28].

The usual method of separating the reversible term from total susceptibility is to apply a reversal at some point on the major hysteresis loop. Provided the amplitude of reversal is small enough to neglect the minor hysteresis loop, the average slope of the reversal magnetization curve is the reversible susceptibility, whereas the slope of the major hysteresis loop determines the total susceptibility [29–33].

The paper is organized as follows. Section 2 briefly reviews the state of knowledge on possible approaches to model reversible magnetization processes in chosen phenomenological models. In this section, three possible extensions to the description based on $T(x)$ transformation [34] are presented. Two of the aforementioned extensions rely on the addition of additional (either linear or nonlinear depending on field strength) terms to the basic $T(x)$ model. The third description, to a large extent, resembles the product Preisach model. The sum of components responsible for the reversible and

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irreversible processes is modulated with a parabolic function of instant magnetization. The proposed ordinary differential equation may be expressed using a closed analytical form. Section 3 presents the modelling results for a sample of grain-oriented electrical steel used in transformer laminations. Section 4 comments on the results.

Hysteresis modelling may be carried out in either the $B = B(H)$ or the $M = M(H)$ coordinate system (H denotes field strength, M stands for magnetization, whereas B is magnetic induction). The representation in the $B = B(H)$ is more useful for electrical engineers, who are more accustomed to working with variables directly related to current and voltage in the windings. From the physicists' perspective, the $M = M(H)$ coordinate system brings more insights on the physics of magnetization processes. At the same time, it should be recalled that magnetization M is a quantity which cannot be directly measured.

In the present paper, we have worked exclusively with the 'internal' hysteresis loops $M = M(H)$, taking into account the audience of the journal. Recalculation of the results may be achieved easily if one accepts the constitutive relationship $B = \mu_0(H + M)$ (we use the Sommerfeld convention and SI units).

2. State of knowledge

Among numerous hysteresis models, the most commonly used in electromagnetism are the Preisach [35], Stoner–Wohlfarth [36] and Jiles–Atherton (JA) [18, 37] descriptions. A brief introduction to contemporary hysteresis models is provided in a recent publication [38].

The Preisach model is a typical 'bottom-up' approach with a sound mathematical background [39, 40]. Hysteresis is described as the result of a cooperative action between a number of elementary units called hysterons. In the classical form, these hysterons have rectangular (relay-like) characteristics. Thus, it is commonly believed that reversible magnetization processes are neglected in the analysis, yet some authors speak of the apparent reversible magnetization concentrated along the diagonal of the Preisach semi-plane [9, 40–42].

The simplest conceptual approach to include reversible processes in modelling related to the Preisach description is to modify the shapes of characteristics of hysterons by introducing an appropriate skewing (play and stop models [43, 44]) or non-linear segments calculated from the Stoner–Wohlfarth (SW) equation [40, 45, 46]. The SW model is an inherently vectorial micro-magnetic model, in which hysteresis for an elementary particle occurs as the outcome of competition between energy contributions due to anisotropy and interactions with the applied field. A magnetization curve is fully reversible if the magnetic field is applied perpendicularly to the easy anisotropy axis. On the other hand, when the magnetic field is applied along the easy axis, the rectangular characteristics are obtained. For intermediate directions, the $M - H$ dependence contains both reversible and irreversible segments.

Della Torre and co-workers developed a number of modifications to the classical Preisach model which were aimed at a better representation of reversible magnetization processes (the product Preisach model, the moving model, the DOK model, etc.) [39, 47–49]. Generally speaking, these extensions rely on either the introduction of some feedback in the block diagram containing the basic Preisach transducer, or on an appropriate modulation of its output [50]. In the present paper, we examine an approach inspired by the product model but using the $T(x)$ description. Previously, a similar modification was proposed for the JA model [51, 52].

The JA formalism was developed in the 1980s in order to study magnetomechanical effects in steels [18, 53]. This phenomenological 'top-down' description incorporates a number of physically justified and plausible concepts (the effective field, the anhysteretic curve, the effects of pinning sites on domain wall movement) and allows one to model hysteresis curves using a set of relatively simple equations, including one ordinary differential equation. The model may be relatively easily implemented numerically for unidirectional magnetization patterns. This justifies its widespread use in the engineering community. Moreover, physical interpretations are attributed to the JA model parameters, which make this description attractive for physicists. However, it should be remarked that for excitation conditions occurring frequently in the work of electrical machines, it is necessary to apply a vector extension of the model [55, 56], which is not as simple to implement as the scalar one is.

The most widely used version of the JA model [37] introduced a decomposition of differential susceptibility into irreversible and reversible components. The subsequent 1992 paper on parameter estimation [54] introduced a slightly different equation for the total susceptibility, which clearly indicates that a sort of weighting is employed in the calculations. Weight coefficient is the model parameter c, which is defined as the ratio of initial normal and anhysteretic susceptibilities.

Most papers devoted to the JA model focus on the following issues: estimation of model parameters using different techniques [14, 57–64], analysis of coupled problems [10, 12, 53, 65, 66] and possibilities of the model to describe magnetization processes in novel materials and structures [3, 58, 67–70]. A number of references report on the necessity to update the values of some model parameters in order to obtain a quantitatively correct representation of minor loops and reversal curves [14, 18, 51, 57, 58, 68, 71, 72]. In the context of the subject of the present paper, it is interesting to note the reported variations of reversibility parameter c upon the changes of stress [10, 12], temperature [65, 66, 73] or applied field [57, 67, 68, 71, 72]. Some authors pointed out the problems with the description of magnetization curves after a sudden field reversal (the necessity to introduce the pseudo-parameter δ_M suppressing the irreversible magnetization term, thus eliminating negative differential susceptibilities) [74–76].

The Takács model is a relatively new model of hysteretic phenomena, based on extensive use of hyperbolic tangent transformation. The concept of using the $\tanh(x)$ function in hysteresis modelling is mentioned for the first time in the textbook by Bozorth [77] and the paper by Bullingham and Bernal [78]. Takács scrutinized the idea and formulated the rules for updating the pseudo-constants occuring in the description of reversal curves and other $M - H$ dependencies of practical importance [34, 79]. It should be remarked that some authors used other mathematical functions instead of $\tanh(x)$, e.g. a combination of $\tanh(x)$ and $\mathrm{sech}(x)$ (referred to as the $A(x)$ model) for power system studies [80] or the unipolar sigmoid transformation $U(x)$ applied to shape memory alloys [81]. The $T(x)$ description alone is usually considered as yielding satisfactory results for most soft magnetic materials, yet some authors report possible improvements due to introduced modifications e.g. Padé approximation for $\tanh(x)$ [82] or additional terms for low field region [83, 84].

2.1. $T(x)$ model with a linear term for the description of irreversible processes

Such an extension is mentioned in the textbook [34]. Takács claimed that the irreversible and reversible processes might be separated and the latter ones might be taken into account by adding a certain linear term to the $T(x)$ function. For example, symmetric hysteresis loops (in dimensionless units) are described with the following set of expressions:

$$y = \tanh(x - \delta a_0) + \delta b + A_1 x \tag{1}$$

$$b = 0.5\left[\tanh(x_m + a_0) - \tanh(x_m - a_0)\right], \tag{2}$$

where a_0 is the reduced coercivity, $\delta = \pm 1$ is the sign of time derivative of control variable ($\mathrm{d}x(t)/\mathrm{d}t$ for the forward model, $\mathrm{d}y(t)/\mathrm{d}t$ for the inverse model), whereas x_m is the coordinate of crossover point for both loop branches. A_1 is an appropriately chosen constant.

2.2. $T(x)$ model with a non-linear term for the description of irreversible processes

Włodarski noticed that under certain circumstances the linear reversible term might produce an excessive system response [85]. In his description, total magnetization was modelled as the sum of two Langevin functions with different slopes. Adapting this line of reasoning, the following expression for symmetric hysteresis loops may be written

$$y = \tanh(x - \delta a_0) + \delta b + A_2\left[\coth x - \frac{1}{x}\right]. \tag{3}$$

A somewhat similar approach was considered by Mészarós, but this author used the $\tanh(x)$ function with an appropriate weight [86].

In order to illustrate the qualitatively different behaviour of both models, a simulation result is shown in figure 1. For both descriptions, the following parameters are chosen $a_0 = 1, A_1 = A_2 = 0.1$. It can be seen that the hysteresis curves obtained with the second approach reveal lower susceptibilities than those modelled with the first description for the same values of parameters.

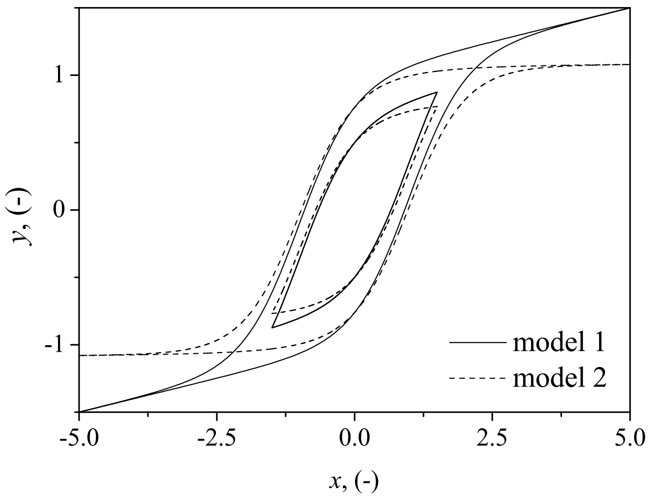

Figure 1. Exemplary modelled hysteresis loops.

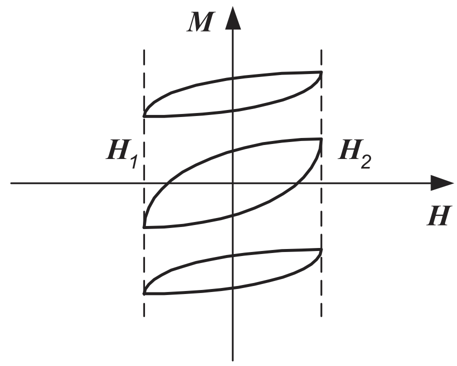

Figure 2. Noncongruency of minor hysteresis loops.

2.3. Product model with $T(x)$ for the description of irreversible processes

Kádár developed the so-called product Preisach model, which was able to address some deficiencies of the classical Preisach model (e.g. zero initial susceptibility) [47, 49]. The idea is to introduce a modulated sum of two components, related to reversible and irreversible processes that may be written as

$$\frac{\mathrm{d}M}{\mathrm{d}H} = R(M)\left[\beta + \int_{H_0}^{h} Q(h, h')\mathrm{d}h'\right], \tag{4}$$

where the limiter function $R(M)$ may be assumed in the first approximation as parabolic, $R(M) = 1 - (M/M_s)^2$, β is the reduced Rayleigh constant, whereas the last term in the bracket is calculated from the classical Preisach model. A similar expression for the total magnetization was proposed by Chwastek in his modification to the JA model [51, 52], yet it should be remarked that in the aforementioned description there existed a coupling between the last term in the bracket and total susceptibility. This coupling was expressed through the so-called effective field, being an indispensable component of the JA model. In the product Preisach model, there is no such coupling, thus the book-keeping procedure describing the actual macroscopic magnetization state is

Table 1. Estimated values of model parameters.

	H_{c0} (A m^{-1})	a (A m^{-1})	$A_1/A_2/\beta$ (—)/(kA m^{-1})/(—)	M_s (A m^{-1})	Resnorm (A m^{-1})^2
Model 1	14.4	7.2	837.5	$1.30 \times 10^{+06}$	$1.0645 \cdot 10^{11}$
Model 2	14.5	8.5	22.7	$1.37 \times 10^{+06}$	$1.4727 \cdot 10^{11}$
Model 3	14.5	8.9	$11.5 \cdot 10^{-3}$	$1.72 \times 10^{+06}$	$1.0452 \cdot 10^{11}$

separated from the statistical switching of hysterons driven by magnetic field only [47]. The possibility to separate magnetization and field related processes is referred to as the state-independent hypothesis [87]. The limiter function $R(M)$ modulates the shape of minor loops introducing their noncongruency (loops with higher average magnetization are flattened)—see figure 2. This feature is characteristic for many soft magnetic materials [88–91]. The parabolic profile of the limiter function $R(M)$ was considered for the first time in 1911 by Gans [92].

By analogy to the expression (4), we propose to model symmetrical hysteresis loops with the following relationship

$$\frac{\mathrm{d}y}{\mathrm{d}x} = R(y)\left[\beta + \frac{\mathrm{d}f(x)}{\mathrm{d}x}\right], \quad (5)$$

where $f(x) = \tanh(x - \delta a_0) + \delta b$. In this way, a hybrid 'product—T(x)' model is created. Notice that expression (5) reduces to the simplest $T(x)$ model, as considered in the textbook [34], if ones takes $R(y) \equiv 1$ and $\beta \approx 0$.

In the subsequent analysis, we assume the parabolic form for the limiter function following the works by Gans and Kádár. The assumption that $R(y) = 1 - y^2$ is equivalent to the introduction of nonlinear modulation applied to total susceptibility. At this point, it should be recalled that some authors interpreted the limiter function as a measure of total domain wall surface area present at various levels of magnetic saturation [49, 93, 94].

An interesting feature of the proposed description is that the differential equation (5) may be solved analytically for the H-input case. Because in the analysis we focus on modeling hysteresis in quasi-static conditions (low excitation frequency means that distortion of magnetization curves due mostly to eddy currents may be neglected [51, 65, 96]), the shapes of hysteresis loops for the H-input and the B-input cases are the same. Closed analytical formulas are usually preferred to numerical solutions.

For $|y| \leqslant 1$, one may write

$$\int_{y_{in}}^{y_{cur}} \frac{\mathrm{d}y}{1 - y^2} = \int_{x_{in}}^{x_{cur}} \beta \mathrm{d}x + \tanh(x - \delta a_0)\Big|_{x_{in}}^{x_{cur}}$$

thus

$$\begin{aligned} \mathrm{atanh}\, y_{cur} - \mathrm{atanh}\, y_{in} &= \beta(x_{cur} - x_{in}) \\ &+ \tanh(x_{cur} - \delta a_0) - \tanh(x_{in} - \delta a_0), \end{aligned} \quad (6)$$

where the subscript 'cur' stands for the current value of x or y, whereas the subscript 'in' denotes the initial value. For a symmetrical loop $x_{in} = -x_m$ and $x_{cur} = x_m$, where the subscript 'm' denotes the value at loop tip.

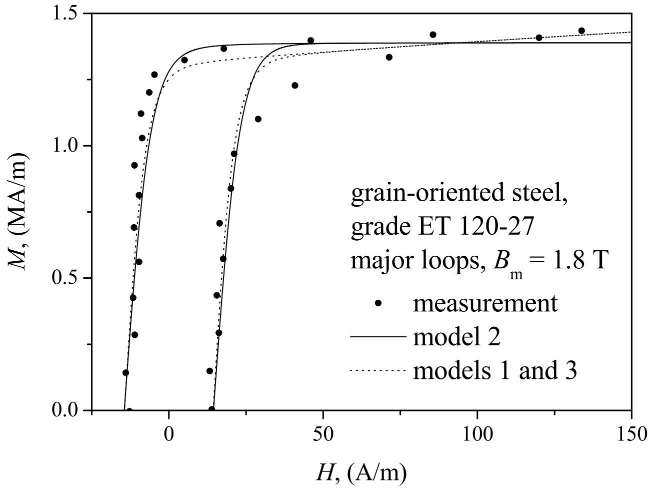

Figure 3. Major hysteresis loops for the grain-oriented steel.

3. Modelling

It follows from the previous section that the $T(x)$ description is purely phenomenological, thus the model users have to resolve what physical meaning is attributed to the variables x and y. In this paper, we assume that x stands for the applied magnetic field strength, whereas y is magnetization, both expressed in (A m^{-1}). Induction (flux density) is related to the aforementioned quantities with the constitutive relationship $B(t) = \mu_0(H(t) + M(t))$. The assumption that x denotes the applied field strength means that mean field effects are neglected. The argument of limiter function $R(y)$ is reduced magnetization (referred to as saturation value). Thus, the basic model equation for the third description (equation (2)) takes the following form in physical units (similar rules apply to models 2 and 3)

$$M = M_s \tanh\left[\frac{H - \delta H_{c0}}{a}\right] + \delta b + A_1 H \quad (7)$$

where

$$b = 0.5 M_s \left[\tanh\left(\frac{H_m + H_{c0}}{a}\right) - \tanh\left(\frac{H_m - H_{c0}}{a}\right)\right] \quad (8)$$

and $\delta = \pm 1$.

For modelling purposes, we used the measurement data pertaining to a grain-oriented 3.2 wt. Si% steel, 0.27 mm thick. The commercial designation of the grade used is ET120-27.

The choice of the material for the study was motivated by the following factors:

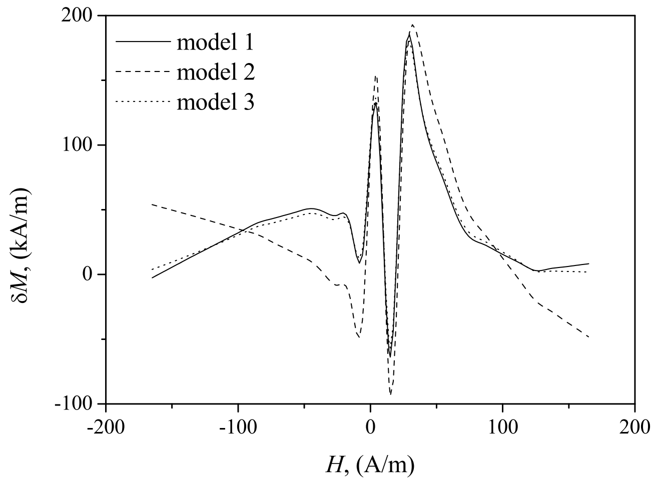

Figure 4. Deviations between the measured and the modelled data points belonging to the ascending branch of major hysteresis loop.

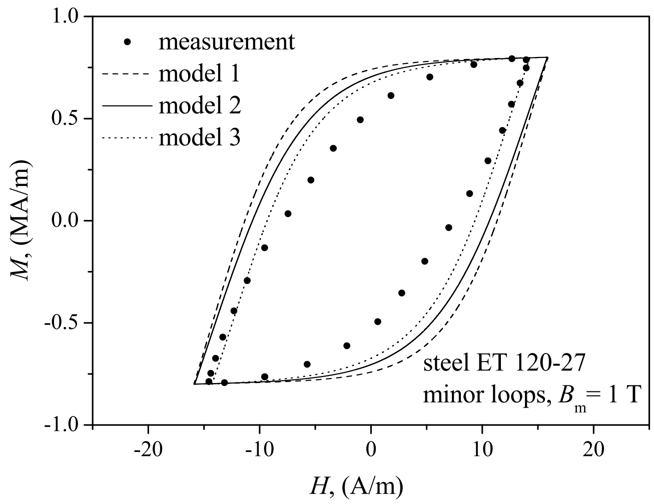

Figure 5. Minor hysteresis loops for the grain-oriented steel, $B_{\mathrm{m}} = 1.0$ T.

- grain-oriented electrical steel covers an important fraction (around 16 %) of the soft magnetic materials used in practical applications [95]; it is the material of choice for electrical engineering working on transformers;
- for grain-oriented steel the hysteresis loop is steep and narrow, thus a description based on the hyperbolic tangent function might be useful for its representation;
- the internal structure of grain-oriented steel consists mainly of Bloch walls. For this highly textured material, the choice of parabolic function $R(m)$ is justified.

The measurements were carried out using a single sheet tester in quasi-static conditions to avoid the side-effects from eddy currents [96]. The model parameters were estimated using the trust-region-reflective algorithm implemented in Matlab procedure, lsqcurvefit. Their values are provided in table 1. The last column in the aforementioned table marked as 'resnorm' denotes the sum of squared errors between the measured and modelled values of magnetization for 34 equidistant data points belonging to the ascending branch of the major hysteresis loop.

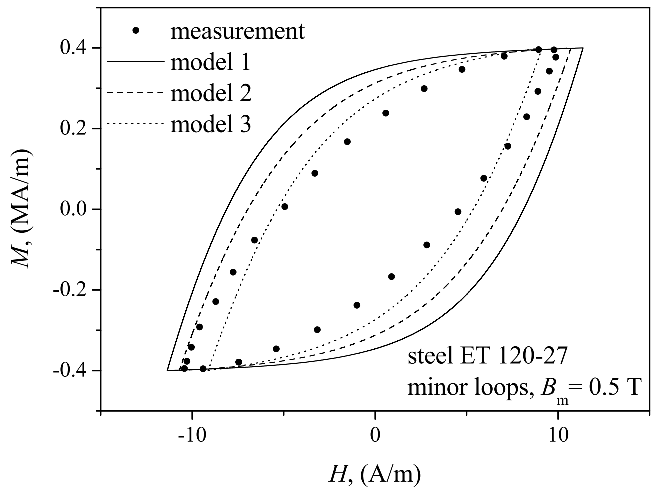

Figure 6. Minor hysteresis loops for the grain-oriented steel, $B_{\mathrm{m}} = 0.5$ T.

It can be stated that the values of H_{c0} and normalization parameter a obtained for all models were quite similar. The measured value of coercive field strength for the major loop (12.7 A m^{-1}) was slightly lower than the estimated values for H_{c0}. For models 1 and 2, the estimates for saturation magnetization were below the actual magnetization value at major loop tip, whereas for model 3 the optimization routine has stuck at the value close to theoretical saturation of iron (2.16 T). During the estimation, no constraints for the estimated variables were introduced.

Figure 3 presents a magnified fragment of the modelled hysteresis loops for $B_{\mathrm{m}} = 1.8$ T. The hysteresis curves are presented using the $M = M(H)$ coordinate system, however, the notation $B_{\mathrm{m}} = 1.8$ T means that the induction amplitude was controlled during measurements.

Figure 4 presents the deviations between measured and modelled data points belonging to the ascending branch of the major hysteresis loop for all considered descriptions. It can be stated that models 1 and 3 offer comparable accuracy, whereas model 2 performs slightly worse.

Figures 5 and 6 depict the modelled hysteresis loops for $B_{\mathrm{m}} = 1.0$ T and $B_{\mathrm{m}} = 0.5$ T, respectively. The measurement points are marked with dots. It can be stated that the third model describes the symmetrical minor loops most accurately.

4. Conclusions

In this paper, three possible approaches to describe reversible magnetization processes are presented in the context of the phenomenological $T(x)$ model, based on hyperbolic tangent mapping. The method yielding best accuracy for minor hysteresis loops of grain-oriented electrical steel used for transformer laminations resembles, to a large extent, the product Preisach model. Total susceptibility is modulated with a magnetization-dependent function assumed as parabolic. The magnetization-dependent limiter function had been interpreted previously by some authors as a measure of total domain wall surface area, therefore, it is felt that the proposed product-$T(x)$ model sheds some light on the physics of magnetization processes. The advanced description is compliant with the so-called state-independent hypothesis. It has been shown that the resulting differential equation may be solved analytically for the H-input case.

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