

Homogenization of Metal Grid Reinforced Composites for Near-Field Low Frequency Magnetic Shielding

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Abstract—The purpose of this paper is to provide simple analytical homogenization methods for composite materials containing a metallic wire grid. Estimating their effective electrical properties facilitates the numerical simulation of composite structures for shielding applications in the automotive industry. The presented methods are based on surface impedance approaches and effective media theory. The obtained results show that the shielding properties of the described wire grid composites can be accurately estimated and bounded, using the proposed theories in the low frequency range. The frequency limits vary according to the studied sample. For the presented materials, the validity of the results is shown to be up to a few megahertz. The experimental validation is done by measuring the shielding effectiveness of composite samples using a near-field test bench.

1. INTRODUCTION

Reducing the environmental impact of modern vehicles can be improved through weight optimization approaches. Therefore, one of the steps that can be undertaken is the replacement of classically used metal alloys with lighter yet conveniently robust composite materials. From an electrical viewpoint, composites formed by placing metallic grids in an epoxy resin can substitute metal structures if they prove capable of providing acceptable electromagnetic shielding levels. Shielding applications in the automotive industry (e.g., shielding enclosures for electrical conversion chains, plates for wireless charging systems of electric vehicles) hold importance in the lower frequency range, usually up to a few megahertz.

Composite materials made of conductive reinforcement and dielectric matrices have been the subject of multiple studies to determine their electrical and shielding properties. While numerically-based approaches offer accurate results in this regard [1–7], the need for simple and fast methods which can be used for conducting preliminary studies makes analytical design tools desirable. When this path is pursued, two types of studies can be adopted. The first aims at predicting effective properties based on equivalent impedance theories [8–12], whereas other homogenization techniques use mixing rules [13–18] to provide estimates or bounds for the effective electrical properties of the material with respect to frequency. Shielding Effectiveness (SE) estimations are then obtained typically using classical transmission-line theory.

In previously published approaches, experimental measurements combined with inverse problem solving techniques allowed to deduce homogenized properties of composite materials. In [19], composite panels for aerospace applications were studied in low frequency, and in [20], different types of carbon fiber reinforced composites for air-vehicles have been considered in the range [100 kHz–18 GHz]. In this paper, simple analytical formulas are provided for metal grid composite materials used in the automotive industry and experimentally validated using a near-field test bench. In the interest of facilitating the

Received 24 May 2020, Accepted 9 August 2020, Scheduled 10 December 2020

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integration of composite structures in numerical simulations, a homogenization approach is adopted. The heterogeneous material is replaced with a fictitious homogeneous medium that has an identical electromagnetic EM response. The work is presented as follows. In the first part, the composite material is described. Two homogenization techniques are then introduced. One is based on the surface impedance method proposed in [21] and implemented in [10] for plane wave excitation. Here however, a low-frequency approximation is presented, and an estimation of the effective conductivity is deduced. The other uses mixing theory to provide upper and lower bounds of the shielding coefficient SE. In the second part, these approaches are applied to two composite samples reinforced with metallic grids. A near-field test bench is used to test their accuracy through multiple configurations and over the frequency band of interest.

2. HOMOGENIZATION METHODS

The aim of this section is to propose adequate methods for the estimation, and bounding, of the effective conductivity of square metal grids immersed in a dielectric material (Fig. 1). The effective permittivity is neglected due to the abundance of conductive currents with respect to displacement currents in the quasi-static frequency range. Hereafter, the homogenization methods are described.

2.1. Surface Impedance Technique

The proposed technique relies on the definition of homogenization which supposes that the existence of a fictitious medium that is electrically equivalent to the heterogeneous composite. According to this hypothesis, there exists a homogeneous conductive plate having the same shielding properties as the square grid. The estimation of the effective conductivity for a quasi-static excitation is based on the following analysis. First, the square grid is studied by cascading two wire arrays. In [21], the complex surface impedance of each array Z_s is given (Equation (1)) as a function of the wire diameter d , cell size p , the electrical properties of both the matrix and fibers (Fig. 1) as well as the impedance per unit length of the conductive wires Z_w taking into account skin depth within the wire [22].

$$Z_s = \left[Z_w p + j \alpha \sqrt{\frac{\mu_m}{\epsilon_m}} \right] \quad (1)$$

$$\text{where : } \begin{cases} \alpha = \frac{d}{\lambda_m} \cdot \log\left(\frac{p}{d}\right) \\ \lambda_m = \frac{C}{f \cdot \sqrt{\epsilon_{rm}}} \end{cases}$$

ϵ_m , μ_m and λ_m are the permittivity, permeability, and wavelength in the dielectric matrix, respectively. $C = 3 \cdot 10^8$ m/s is the wave velocity in vacuum, and f is the excitation frequency.

Then, the case of a plane wave excitation is considered as in [10]. For the material at hand, applying the described cascading technique, when the wave is normally incident and x -polarized (Fig. 1), gives the total impedance of the plate $Z_{sT} = Z_s$. In the quasi-static case, the plate's resistivity R_s (represented by the real part of the impedance) is dominant. It corresponds to that of an equivalent homogeneous plate defined by its effective low frequency conductivity $\tilde{\sigma}$ as in Eq. (2).

$$\tilde{\sigma} = \frac{1}{R_s \cdot 2d} = \frac{1}{\text{real}(Z_{sT}) \cdot 2d} \quad (2)$$

It is worth noting that the considered thickness of the equivalent plate is that of the wire grid. Hence, the term $2d$ is used to represent it in Eq. (2).

The above formula, while simple, is only valid for square grids where the material is considered isotropic in the (xy) plane. Its limits, frequency wise, are to be studied on a case-by-case basis since the quasi-static threshold is dependent on the grid's physical and electrical properties. This is shown in the experimental validation section.

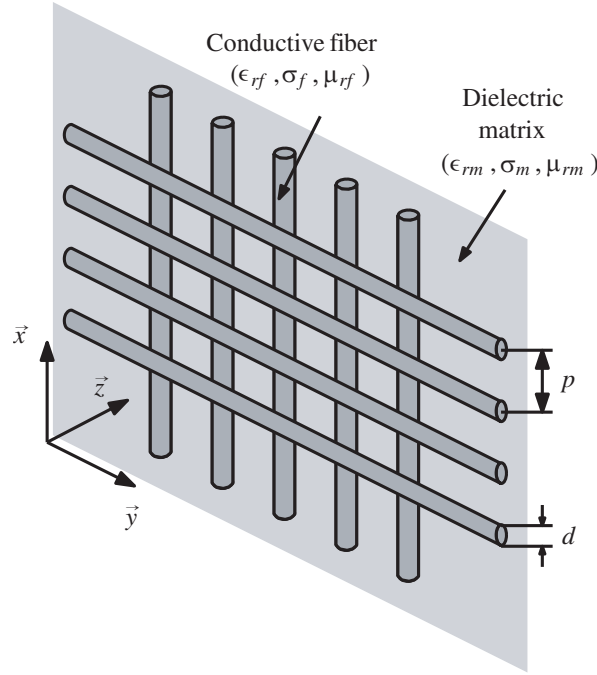


Figure 1. Composite material: a square grid of conductive fibers and a dielectric matrix.

2.2. Mixing Theory Method

In the previous section, an estimation of the effective conductivity was presented. However, it is also helpful to provide lower and upper bounds of this property. This gives an idea of the range of shielding levels that can be obtained from the composite since shielding effectiveness of isotropic media is directly related to their conductivity. The search for a suitable homogenization method is limited by a multitude of parameters. The frequencies of interest fall perfectly within the application domain of classical analytical mixing rules. Moreover, the macroscopic isotropy of the material as well as the low volume fraction of fibers and excitation polarization reasonably reduce the solution to the Hashin-Shtrikman HS bounds (σ_{HS-} and σ_{HS+}) [23]. Thus, the effective conductivity $\tilde{\sigma}$ of the composite can be bounded as follows:

$$\sigma_{HS-} \leq \tilde{\sigma} \leq \sigma_{HS+} \tag{3}$$

where (for $\sigma_m < \sigma_f$):

$$\begin{cases} \sigma_{HS-} = \sigma_m + \frac{f_v}{\frac{1}{\sigma_f - \sigma_m} + \frac{(1 - f_v)}{2 \cdot \sigma_m}} \\ \sigma_{HS+} = \sigma_f + \frac{(1 - f_v)}{\frac{1}{\sigma_m - \sigma_f} + \frac{f_v}{2 \cdot \sigma_f}} \end{cases} \tag{4}$$

σ_m and σ_f represent the conductivity of the matrix and the fibers respectively. f_v is the volume fraction of the fiber component in the material. The accuracy of this method is tested in the experimental validation part.

3. EXPERIMENTAL VALIDATION

In the previous section, two homogenization processes were presented. In order to test their validity with respect to the application, i.e., magnetic shielding in the quasi-static frequency band, a shielding coefficient must be estimated from the defined effective medium. The results should also be tested

experimentally. Therefore, in this part, analytical formulas and numerical models capable of computing the near-field magnetic shielding effectiveness are presented. Their accuracy and the accuracy of the effective conductivity estimates are evaluated using an experimental test bench.

3.1. Near-Field Test Bench

In general, multiple experimental setups can be used to measure electromagnetic shielding effectiveness [24–26]. However, this paper targets shielding applications in the automotive industry where sources of perturbation are predominantly magnetic and wave propagation in the near-field is of interest. To experimentally validate the proposed homogenization methods, and taking these conditions into consideration, the near-field test bench [27, 28] of Fig. 2 is used to evaluate the shielding effectiveness of two wire grid composite samples. The bench is made of two modules. The transmitting chain uses a low frequency signal generator and a coil to generate a magnetic field. The receiver and acquisition module connects a magnetic field sensor (AARONIA AG 12mm: PBS H2) to a spectrum analyzer (Tektronix RSA5106A) through a low-noise amplifier. The captured information is then transmitted to a computer for processing. Once the position of the probe is fixed by controlling the robot arm attached to it, a two-step measurement process is carried out where the magnetic field levels are measured with, then without, the sample. The magnetic shielding effectiveness is defined as the variation introduced in the magnetic field magnitude (in dB) when the shielding material is placed between the magnetic source and the probe. It should be mentioned that the near-field test bench used in this paper is not set in a controlled test environment. Background noise is present during the measurements. Their levels are monitored using the spectrum analyzer to ensure an acceptable signal to noise ratio. Nevertheless, in the presence of the shield, the higher the frequency is, the lower the power of the received signal is. This makes inaccuracies highly probable at higher frequencies. Adding an amplifier to the transmitting chain could provide a solution to this problem.

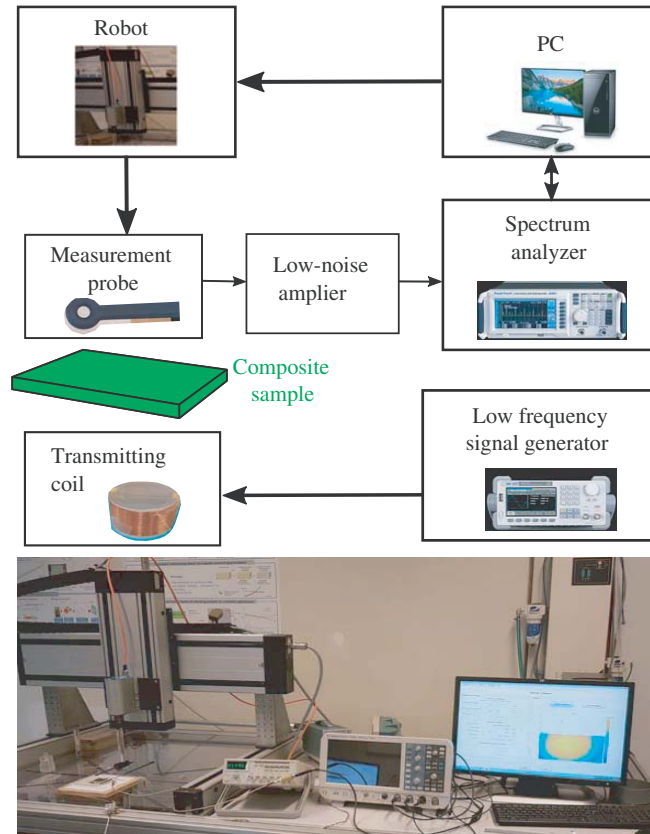


Figure 2. Near-field test bench.

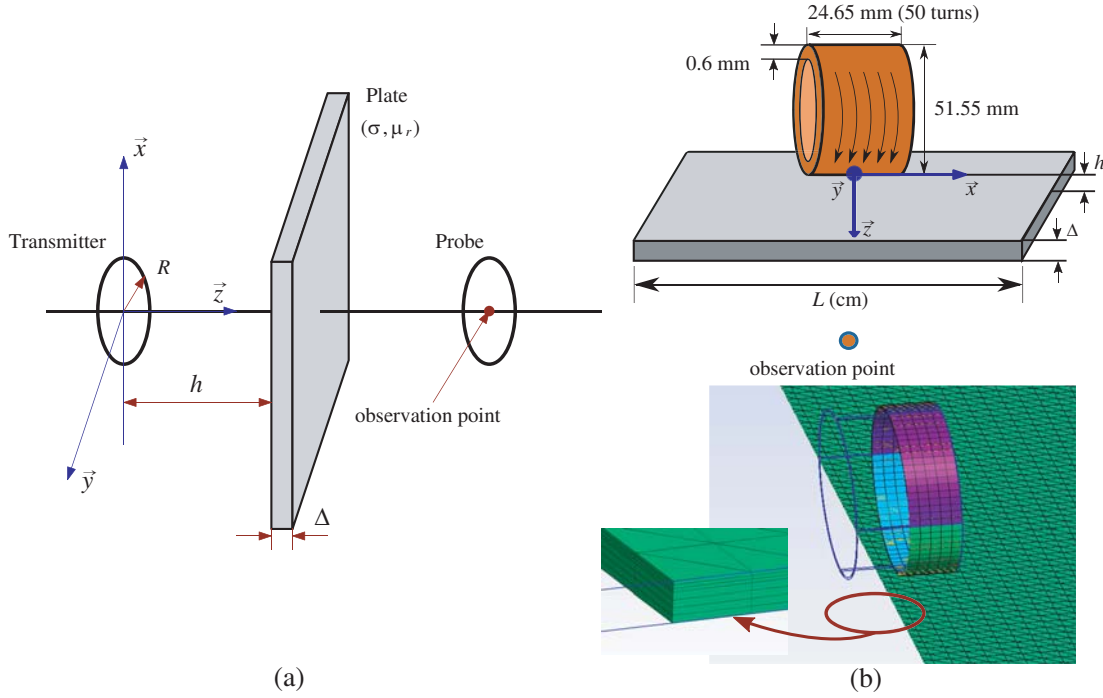


Figure 3. Magnetic shielding setup: (a) parallel and (b) perpendicular configurations.

3.2. Near-Field Magnetic Shielding Effectiveness

Homogenization methods generate effective material properties. Testing the validity of the proposed approaches requires an estimation of the corresponding shielding effectiveness. Hereinafter, an analytical and a numerical method are introduced to provide estimates of shielding properties of homogeneous isotropic plates under two configurations. An analytical formula derived in [29, 30] is used for the case where the emitting coil is parallel to the plate (Fig. 3(a)). It is worth noting that the parallel configuration formula was derived for a circular loop source; however, its results (see next section) will prove to be equally accurate for the cylindrical coil used in the measurements, which is described in Fig. 3(b)). For the perpendicular configuration, the analytical results are not as precise. Thus, a Finite Element Analysis FEA using GMSH/GetDP [31, 32] is carried out for this configuration.

Composite materials reinforced with wire grids usually exhibit anisotropic behaviors. In the case of a square grid, the material is isotropic in the plane of the grid: (xy) plane of Fig. 1. Consequently, the proposed homogenization reduces the effective conductivity to a scalar value. The parallel configuration test, on its own, does not assert the validity of this assumption, hence the need for a second test capable of confirming the ‘isotropic effective property’ hypothesis. The perpendicular configuration test is used for this purpose.

Magnetic shielding effectiveness $SE_{H,dB}$ is defined as the ratio of the magnetic field at an observation point (Fig. 3) without the sample $|H_{w/o}|$ to the transmitted field with the sample $|H_w|$. Expressed in dB, it is written as:

$$SE_{H,dB} = 20 \log_{10} \frac{|H_{w/o}|}{|H_w|} \tag{5}$$

While Eq. (5) is a general definition, for the parallel configuration of Fig. 3(a), the shielding effectiveness on the symmetry axis of a loop illuminating a plane shield of infinite extent is computed analytically as follows [29]:

$$SE_H = \left| \frac{1}{4\mu_r} \frac{\int_0^{+\infty} \lambda^2 \tau_0^{-1} \mathcal{J}_1(\lambda R) e^{-\tau_0 z} d\lambda}{\int_0^{+\infty} K \lambda^2 \tau \tau_0^{-2} \mathcal{J}_1(\lambda R) e^{-\tau_0 z - (\tau - \tau_0) \Delta} d\lambda} \right| \tag{6}$$

where:

$$\left\{ \begin{array}{l} K = \left[\left(\frac{\tau}{\tau_0} + \mu_r \right)^2 - \left(\frac{\tau}{\tau_0} - \mu_r \right)^2 e^{-2\tau\Delta} \right]^{-1} \\ \tau = \sqrt{\lambda^2 - \gamma^2} \\ \tau_0 = \sqrt{\lambda^2 - \gamma_0^2} \\ \gamma = \sqrt{j\omega\mu_0\mu_r\sigma} \\ \gamma_0 = j\frac{\omega}{C} \end{array} \right.$$

\mathcal{J}_α is the Bessel function of first kind of order α , γ the propagation constant in the shield (in m^{-1}), γ_0 the free-space propagation constant (in m^{-1}), ω the angular frequency (in $\text{rad}\cdot\text{s}^{-1}$), C the free-space velocity of light (in m/s), Δ the shield thickness (in m), R the loop radius (in m), z the distance from the emitting loop to the probe (in m), $\mu_0 = 4\pi \cdot 10^{-7} \text{ H/m}$ the free space permeability, and (σ, μ_r) denote the electrical conductivity and the relative magnetic permeability of the shield, respectively.

As for the perpendicular configuration, the magnetic shielding effectiveness is evaluated by numerically solving an Eddy current problem using a FEA where displacement currents are neglected (this same hypothesis is made for the analytical solution of the parallel configuration). Here, a good mesh of the shield is crucial for obtaining accurate results especially when the skin depth becomes smaller than the thickness of the plate. The mesh of the shielding material takes into account the skin depth, which is a function of the frequency, in the following manner. Two uniformly distributed sets of hexahedral mesh elements are used along the thickness direction of the plate (z -axis). The skin depth is represented using three mesh elements. The remaining part is meshed using four elements.

To test these methods, the analytical method of Equation (6) and the FEA are carried out for an aluminum and a copper sheet, respectively. Table 1 shows the two setups parameters and Fig. 4 the shielding effectiveness with respect to frequency. The good agreement obtained between analytical/numerical and experimental results proves the accuracy of the implemented methods. They can therefore be used to study the validity of the homogenization formulas.

Table 1. Configuration parameters for two metal shields.

Sample	Aluminum	Copper	
Configuration	parallel	perpendicular	
Parameter	Value		Unit
Distance source — shield h	10	12	mm
Distance shield — probe	20	12	mm
Conductivity σ	$3.77 \cdot 10^7$	$5.9 \cdot 10^7$	S/m
Shield thickness Δ	1	0.034	mm
Shield dimensions L	80×80	80×80	cm

3.3. Sample Description, Results and Discussion

In this section, the homogenization of two composite samples made of copper (sample A) and steel (sample B) wire grids is carried out according to the presented methods. The metallic grids are embedded within an epoxy resin using a Sheet Moulding Compound SMC manufacturing process. The estimated effective properties are shown in Table 2 along with the dimensions and electrical properties of the composites. Based on the homogenization results, the shielding effectiveness is computed as a function of frequency when the radiation source is parallel (Fig. 5) and perpendicular to the sample (Fig. 6). Compared to measurement results, the obtained estimations show good accuracy up to a

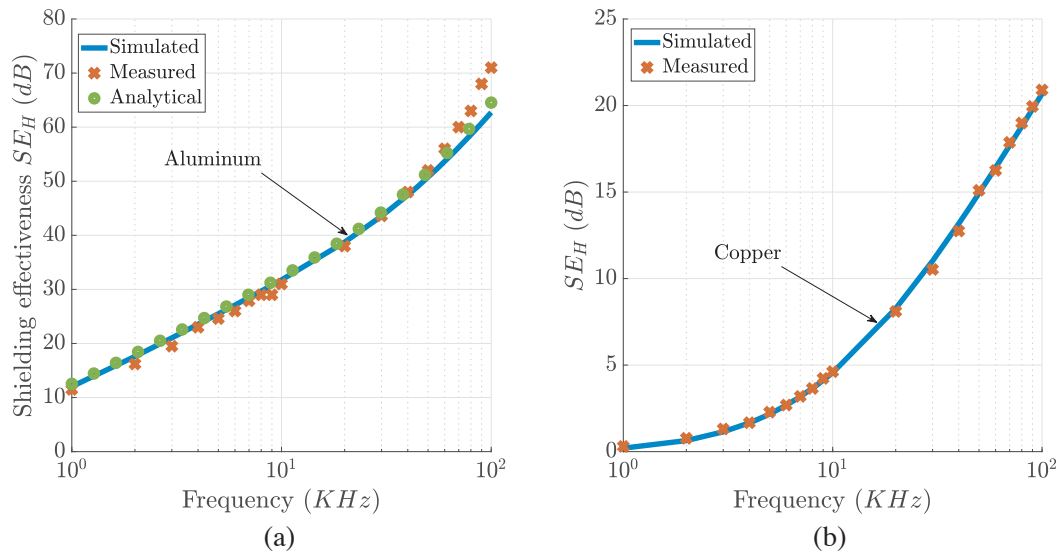


Figure 4. Shielding effectiveness of two metal shields: (a) aluminum according to a parallel setup and (b) copper according to a perpendicular one. Configuration parameters are in Table 1.

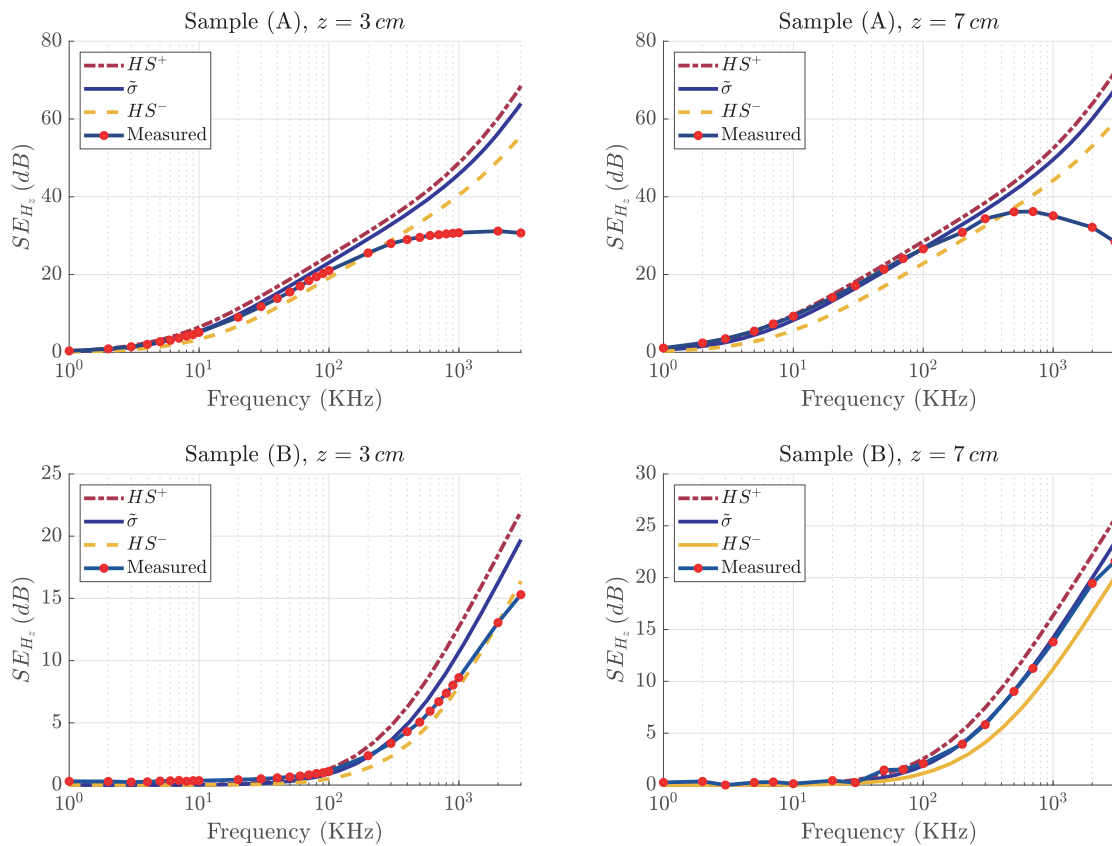


Figure 5. Magnetic shielding effectiveness of composite samples (A) and (B) obtained by homogenization and measurement. A parallel setup (Fig. 3) is considered where the distance h between the source and the shield is 1.5 cm and that between the source and the probe is $z = \{3, 7\}$ cm. Composite parameters are shown in Table 2.

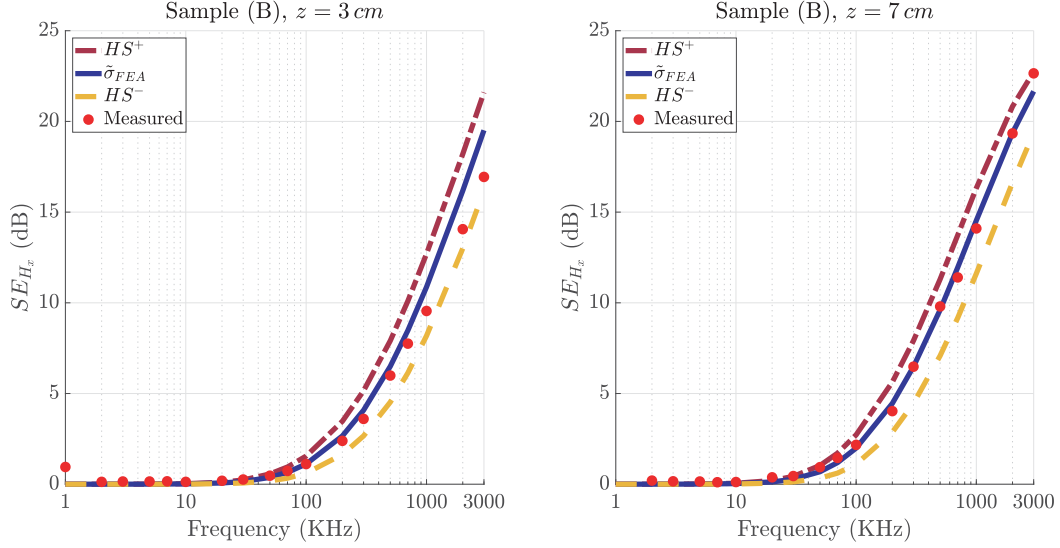


Figure 6. Magnetic shielding effectiveness of composite sample (B) obtained by homogenization and measurement. A perpendicular setup (Fig. 3) is considered where the distance h between the source and the shield is 1.5 cm and that between the source and the probe is $z = \{3, 7\}$ cm. Composite parameters are shown in Table 2.

Table 2. Dimensions and properties of composite samples.

Parameter	Sample (A)	Sample (B)	Unit
	copper grid	steel grid	
Thickness Δ	2	2	mm
Wire diameter d	270	275	μm
Wire conductivity σ_f	$5.84 \cdot 10^7$	$1.4 \cdot 10^6$	S/m
Cell size p	1.32×1.32	1.476×1.476	mm^2
Effective conductivity $\tilde{\sigma}$	$3.89 \cdot 10^6$	$8.68 \cdot 10^4$	S/m
Lower HS bound σ_{HS-}	$2.4 \cdot 10^6$	$5.9 \cdot 10^4$	S/m
Upper HS bound σ_{HS+}	$4.6 \cdot 10^6$	$1.13 \cdot 10^5$	S/m

certain frequency above which a deviation is observed (around 100 kHz for sample A and 800 kHz for sample B). This could be due to a number of reasons, mainly the low-frequency assumption which sets a limit to the validity of the approach. This upper frequency limit is a function of the composite properties and dimensions. Moreover, the accuracy of the measurements decreases with frequency. This is caused by the edge effects that depend on the dimensions of the sample [19].

In [25], it is shown that different electromagnetic shielding measurement methods can yield different results. This statement is equally valid in the case of the studied wire grids. Concretely, shielding effectiveness is closely related to wave-polarization with respect to wire-direction. For each configuration of the near-field setup, the source (coil antenna) radiates a unique magnetic field. The polarization of the field with respect to the wires is specific to the setup of the coil with respect to the composite sample. Hence, the obtained magnetic shielding results are specific to the setup. However, effective properties resulting from the homogenization method are valid, no matter the polarization of the incident wave. This is proven by measuring two different configurations: one where the coil is parallel to the shield and the other where it is perpendicular. The validity of the isotropic effective medium hypothesis is also

proven in this way, since a good agreement between theoretical and experimental approaches is obtained for both the parallel and perpendicular configurations. In this regard, it is worth noting that for the parallel setup, only the z component of the H -field is measured (both x and y components are negligible at the observation point). On the other hand, for the perpendicular setup, only the x -component is measured (Fig. 3).

In the low frequency range, the two approaches provide accurate results: the surface impedance based homogenization technique accurately estimates effective conductivity, and the Hashin-Shtrikman formulas provide very narrow bounds for shielding properties.

4. CONCLUSION

In this paper, simple homogenization formulas for composite materials reinforced with square metal grids are provided. Their application is valid at low frequencies. This can be helpful for the design of composite electromagnetic shields in the automotive industry. Two complementary approaches — estimates and bounds — are presented. An estimate of the quasi-static effective conductivity is computed based on surface impedance studies, and lower and upper bounds of this property are predicted using Hashin-Shtrikman theory (mixing rules). The obtained results indicate that the surface impedance approach accurately predicts low-frequency effective conductivity and magnetic shielding properties of conductive square grids. Moreover, Hashin-Shtrikman formulas are shown to yield very narrow frames of these properties. Their validity, however, is limited to materials with low volume fractions of inclusions (around 8% for the tested specimens). Shielding effectiveness is obtained from the effective properties through an analytical formula and/or Finite Element Analysis. The proposed approaches are validated experimentally using a magnetic near-field test bench.

ACKNOWLEDGMENT

This work is supported by the Automotive Mechatronics Chair, a cooperation between Faurecia, CentraleSupélec and Esigelec.

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