Effective Electromagnetic Properties of Woven Fiber Composites for Shielding Applications

Ghida Al Achkar, Lionel Pichon, Laurent Daniel, and Nabil Benjelloun

***Abstract*—Composite materials reinforced with woven conductive fibers can be good candidates for electromagnetic shielding applications. Their light weight provides an important advantage over metallic alloys classically used in the automotive and aircraft industries. However, numerical modeling of composite-based large-scale structures, such as shielding enclosures, is rendered almost impossible by the heterogeneities at the microscopic scale. This problem is commonly addressed using electromagnetic homogenization methods. In this paper, we propose a homogenization technique based on finite element computations and inverse problem solving for estimating the effective properties of woven composites. The effect of electrical contacts between the fibers is also studied and its influence on the global behavior of the material is analyzed. The results are then shown to diverge from those obtained using analytical mixing rules over the frequency range of 1–40 GHz. Thus the suitability of these homogenization methods is discussed with respect to the studied frequency range and the nature of the microstructure.**

***Index Terms*—Composite materials, effective media, electrical contact, homogenization, optimization, shielding effectiveness (SE), three-dimensional (3-D) finite element method (FEM).**

I. Introduction

THE increased use of composite materials in the automotive and aircraft industries is mainly due to their mechanical properties and reduced weight. However, the electrical properties of certain types of composites can make them good candidates for replacing metal alloys in electromagnetic (EM) compatibility applications. Studying the EM response of composite materials in the vicinity of electronic devices becomes necessary, especially for incorporating such materials in the design of large-scale structures such as shielding enclosures. Modeling these structures with typical numerical methods [finite element method (FEM), finite difference time domain (FDTD)] is nonetheless computationally heavy even when the enclosure is made of homogeneous conductive materials (i.e., aluminum alloys). Implementing composite-based structure models using numerical methods becomes impossible because of the heterogeneous nature of composite materials at the microscopic scale. It is useful to use homogenization methods as solutions for this computational problem. Once a homogeneous equivalent medium for the composite is defined through its complex permittivity and permeability the problem is reduced to solving a standard case of shielding structure.

This idea constitutes a good starting point for the study of woven fiber composites, a material that is widely used, and manufactured through well known processes. The complexity of the geometry as well as the anisotropy of the material make it necessary to define a proper homogenization technique. To this end, a study of the literature shows that numerous EM homogenization methods can be used. They are grouped under three main categories: analytical models, numerical methods, and experimental setups.

Analytical mixing rules can either provide lower and upper bounds of the effective properties, such as Wiener (W) and Hashin–Shtrikman bounds [2], or estimates of the EM properties under certain assumptions regarding the material and excitation (Maxwell-Garnett, Bruggeman, ...) [1]. Their application domain can vary according to a multitude of parameters, including but not limited to frequency, filling factor, and polarization of the EM wave with respect to the heterogeneities or the number of constituents. These formulas have been used to estimate the effective EM properties of various composites over different frequency ranges [3]–[7]. While these analytical mixing rules are fast and efficient, their suitability must each time be verified numerically or experimentally.

As for numerical methods, they are most helpful at the design stage as they allow for parametric examinations of different constituent properties and dimensions. They can however be computationally heavy especially for three-dimensional (3-D) models. Different numerical methods can be used to study the EM behavior of composite materials. At the microscopic scale, the homogenization problem relies on the FEM to extract the average electrical properties [8]–[10]. At the mesoscopic scale, which is that of the woven fabric, numerical methods are used either to compute the effective properties directly, or through EM parameters that are closely related to them, such as the transmission and reflection coefficients. More concretely, computational methods, such as rigorous coupled wave analysis [11] and full wave expansion [12] are not well suited for electrically

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conductive materials. On the other hand, methods such as FDTD [13], [14] can be used to model woven carbon fiber-reinforced polymers by assuming a perfect conductor medium, or also to model thin composite panels. Additionally, other numerical techniques [15], [16] such as circuital approaches and transfer matrix method can be used to compute the effective properties of woven fabrics and metallic wire grids over different frequency ranges.

In this paper, the 3-D problem of modeling woven fiber composites is studied using the FEM. The complexity of the geometry, as well as the skin effect to be considered in the gigahertz band makes this method a suitable one. As for the extraction of the effective properties, experimental techniques [17], [18] have been combined with appropriate inverse problem solving to provide a good estimate of the effective properties of conductive fiber composite materials. However, in this paper a simulation-based homogenization technique is proposed. It allows for a practical study of the contribution of different parameters (fiber contact, conductivity) to the woven material's behavior. Additionally, it is helpful for studying nonsymmetric composites as the polarization of the incident wave with respect to the inclusion can be predefined. In the first part, this technique based on finite element computations and inverse problem solving is presented. In a previous work [19], a simple technique was introduced. Based only on the shielding coefficient, it does not effectively capture the material behavior because it does not take into consideration the reflection part of its response. Moreover it was applied to a 2-D model of a unidirectional fiber composite. Hereinafter, this method is extended for a comprehensive description of the material. It is also applied to a 3-D model of a woven fabric composite that takes into account electrical contacts between the fibers. In the second part, a simplified model of the material is presented in an attempt to explore the applicability of analytical formulas for the structure at hand. In the last part the obtained results are analyzed and a conclusion is drawn.

II. PROPOSED HOMOGENIZATION METHOD GENERAL OUTLINE

The estimation of materials EM properties involves the evaluation of an intermediate parameter from which these properties can be extracted. In our case, the shielding coefficient [20] [shielding effectiveness (SE) SE_{dB}] and the reflection coefficient R_{dB} will be used to extract the effective electrical properties. This method relies on the following two main steps (Fig. 1).

1) (Step 1) Computing $\mathrm{SE}_{\mathrm{dB}}$ and R_{dB} using FEM: By carrying out the simulations detailed in Section III. the plane-wave shielding and reflection coefficients of an infinite composite sheet can be evaluated over a broadband of frequencies.
2) (Step 2) Solving an inverse problem to obtain the effective properties: Once the simulated coefficients for the heterogeneous sheet are obtained from Step 1, a definition of EM homogenization is then used to extract the composite effective complex permittivity. The principle of homogenization being to replace a heterogeneous plate with its homogeneous equivalent, the logic behind the chosen technique is as follows: for each frequency of the incident wave, there exists a pair of effective permittivity and conductivity $(\epsilon = \epsilon_{\mathrm{eff}}; \sigma = \sigma_{\mathrm{eff}})$ associated with a fictitious homogeneous electrical medium that will give the same shielding and reflection coefficients as those obtained for the composite material simulated numerically. Moreover, $\mathrm{SE}_{\mathrm{dB}}$ and R_{dB} are a function of these effective properties $(\epsilon_{\mathrm{eff}}; \sigma_{\mathrm{eff}})$ and other known parameters as shown in (2)–(6). As a result, finding the effective properties means minimizing the cost function F_c

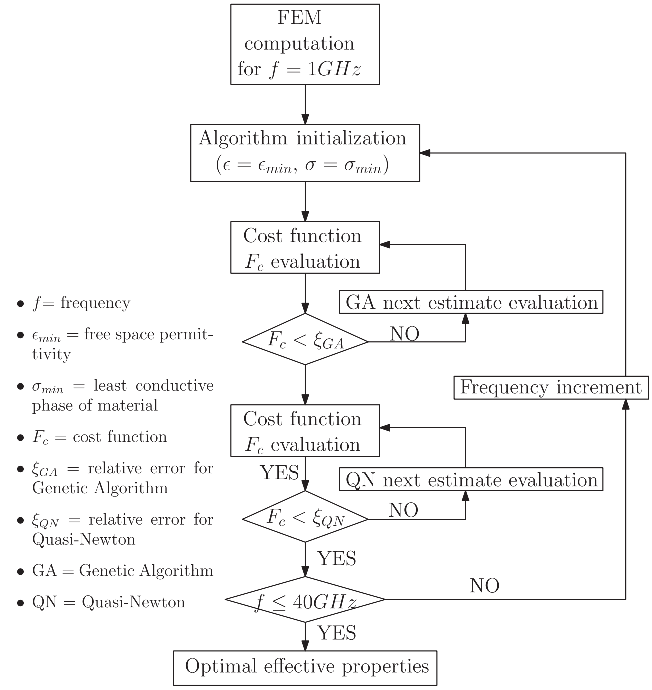

Fig. 1. General scheme of the proposed homogenization technique.

$$F_c = a\,\left|\mathrm{SE}_{\mathrm{FEM}}(f) - \mathrm{SE}_{\mathrm{analytical}}\big(\epsilon(f), \sigma(f)\big)\right|^2_{\mathrm{dB}} + b\,\left|R_{\mathrm{FEM}}(f) - R_{\mathrm{analytical}}\big(\epsilon(f), \sigma(f)\big)\right|^2_{\mathrm{dB}} \qquad (1)$$

where $\mathrm{SE}_{\mathrm{FEM}}$, R_{FEM}, $\mathrm{SE}_{\mathrm{analytical}}$, and $R_{\mathrm{analytical}}$ are the FEM and analytical estimates of the shielding and reflection coefficients. ϵ is the electrical permittivity, σ is the electrical conductivity, and f is the frequency of the incident EM wave. The cost function is written in terms of both the reflection coefficient and the SE. Indeed, in [19], the optimization problem is formulated in terms of only SE. However, an in-depth examination of the previous results showed that they did not adequately fit the reflection coefficient. Moreover, in order to get the best results out of the minimization study, two weighing coefficients a and b are added. This type of regularization shows to give a good estimate of the minimum for the values: $a(f) = (1,\ |R_{\mathrm{FEM}}(f)|)$, $b(f) = (1,\ |\mathrm{SE}_{\mathrm{FEM}}(f)|)$. Choosing a and b's values remains to be determined according to the studied material.

Expressed in dB, the SE, $\mathrm{SE}_{\mathrm{dB}} = \mathrm{SE}_{\mathrm{analytical}}$ of a homogeneous infinite sheet subjected to a normally incident plane wave groups three main phenomena: the absorption of the material

SE_A, the reflection part SE_R and the multiple reflections that occur inside the plate SE_B [21]

$$\mathrm{SE}_{\mathrm{dB}} = \mathrm{SE}_A + \mathrm{SE}_B - \mathrm{SE}_R \tag{2}$$

where

$$\begin{cases} \mathrm{SE}_A &= 20\log_{10}\left(|e^{\jmath kl}|\right) \\ \mathrm{SE}_B &= 20\log_{10}\left(|1 - qe^{\jmath 2kl}|\right) \\ \mathrm{SE}_R &= 20\log_{10}(|g|) \end{cases} \tag{3}$$

with $k = \sqrt{\epsilon\mu\omega^2 - \jmath\mu\sigma\omega}$ the wave vector inside the plate. (ϵ, σ) are the permittivity and conductivity of the plate. l is the plate thickness and ω the angular frequency of the propagating wave. The parameters g and q are functions of the EM properties of the plate

$$\begin{cases} g &= \dfrac{4\eta\eta_0\mu_r}{(\eta_0 + \eta\mu_r)} \\ q &= \left(\dfrac{\eta_0 - \eta\mu_r}{\eta_0 + \eta\mu_r}\right)^2 \end{cases} \tag{4}$$

with $\eta = \frac{\mu_0\omega}{k}$ the impedance of the plate and μ_r the relative permeability. The reflection coefficient R_{dB} is defined as the ratio of the reflected wave to the incident wave

$$R_{\mathrm{dB}} = 20\log_{10}\frac{|\boldsymbol{E_R}|}{|\boldsymbol{E_I}|} \tag{5}$$

which when developed gives

$$R_{\mathrm{dB}} = 20\log_{10}\frac{\left(\eta_0 - \eta\right)\left(e^{\jmath 2kl} - 1\right)}{\left(\eta_0 + \eta\right)\left(q - e^{\jmath 2kl}\right)}. \tag{6}$$

These equations are used in the minimization process in order to extract the effective properties of the composite. As for the optimization algorithm, to ensure a fast convergence, a combination of a genetic [22] and a deterministic algorithm is used. Genetic algorithms are well known for their repeated modification of the population of individual solutions thus avoiding local minima. This algorithm is used to roughly localize the global minimum within a certain error margin, a deterministic quasi-Newton algorithm is then carried out from the stopping point of the previous algorithm for faster convergence towards the global minimum of the cost function.

In the next part, the behavior of woven fiber composites is studied through finite element computations. The obtained shielding and reflection coefficients are used to compute the effective properties with respect to frequency.

III. EM Behavior and Homogenization of Woven Fiber Composites

A. Finite Element Computations

Shielding properties of materials are estimated through the ratio of the incident wave $|\boldsymbol{E_I}|$ to the transmitted wave $|\boldsymbol{E_T}|$, called SE, and defined as follows:

$$\mathrm{SE}_{\mathrm{dB}} = 20\log_{10}\frac{|\boldsymbol{E_I}|}{|\boldsymbol{E_T}|}. \tag{7}$$

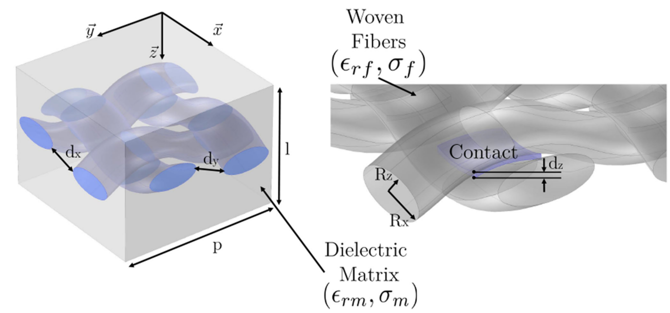

Fig. 2. Woven fiber composite.

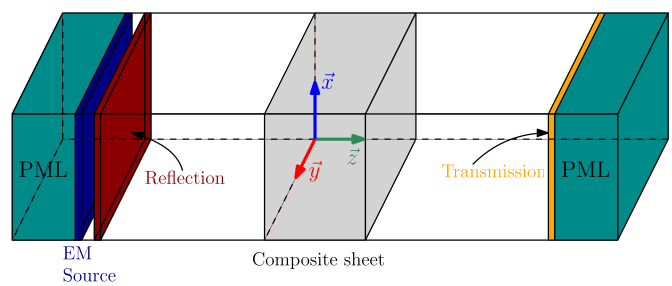

Fig. 3. Three-dimensional finite element computation domain.

A periodic cell of the studied composite is shown in Fig. 2, it contains a plain weave fabric made of conductive fibers drowned in a layer of dielectric material. In order to compute the shielding effectiveness of this geometrically complex material, a finite element approach is adopted using the commercial software COMSOL. The computation domain (Fig. 3) contains a 3-D unit cell of the material surrounded by two layers of air. Two perfectly matched layers are added to prevent undesirable reflections by simulating an infinite medium around the domain [23]. The sheet is illuminated by a plane wave $(\boldsymbol{E_I})$ polarized in the $\boldsymbol{u_x}$ direction

$$\boldsymbol{E_I}(z,t) = E_I\, e^{(\jmath\omega t - k_0 z)}\boldsymbol{u_x} \tag{8}$$

with $k_0 = \frac{\omega}{c}$ the free space wavenumber ($c = 3.10^8$ m.s^{-1}) and ω the angular frequency of the propagating wave.

Frequency domain simulations provide an estimate of shielding effectiveness. At first, the validity of the FEM model is verified by matching the simulated shielding effectiveness of a homogeneous sheet to that obtained by applying the analytical formula detailed in (2)–(6). A sample of the material is then studied, the results represented in Fig. 4 show a pattern of behavior for different values of fiber conductivity σ_f. The nonmagnetic composite essentially acts as a high-pass filter ensuring acceptable levels of attenuation in the lower frequency range.

In [16], SE for different metallic wire meshes is measured. In order to validate the finite element model presented here, a simulation of the woven fabric (sample S1 in [16]) with the following dimensions and properties has been conducted: $l = 270\,\mu$m, $p = 1.116$ mm, $d_x = d_y = 508\,\mu$m, $d_z = 15\,\mu$m, $R_x = 25\,\mu$m, $R_z = 25\,\mu$m, $(\epsilon_{rf}, \sigma_f) = (1, 5.8 \cdot 10^7$ S/m), $(\epsilon_{rm}, \sigma_m) = (3.4, 50 \cdot 10^{-9}$ S/m). It should be noted

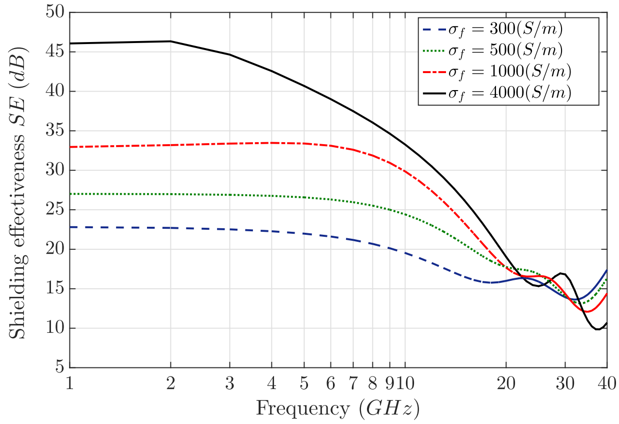

Fig. 4. SE of an infinite sheet (woven fiber microstructure of Fig. 2) for various fiber conductivities σ_f. Configuration: electric field polarized in the $\boldsymbol{x}$ direction, fiber concentration ratio $f_v = 17.5\%$, $l = 3\,\mathrm{m}m$, $p = 4$ mm, $d_x = d_y = 1.22\,\mathrm{mm}$, $d_z = 140\,\mu\mathrm{m}$, $R_x = 625\,\mu\mathrm{m}$, $R_z = 250\,\mu\mathrm{m}$, $\epsilon_{rf} = 1$, $(\epsilon_{rm}, \sigma_m) = (5, 0\,\mathrm{S/m})$.

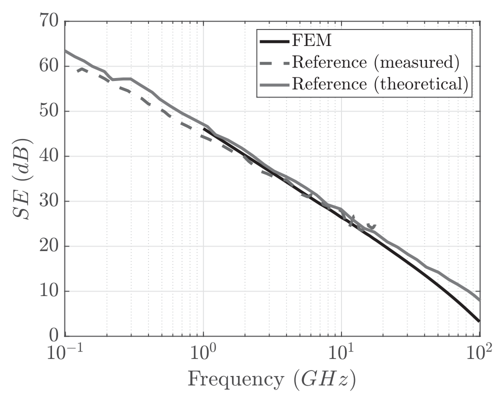

Fig. 5. SE of an infinite sheet (woven fiber microstructure of Fig. 2) compared to simulated and measured data obtained from [16] (sample S1). Configuration: electric field polarized in the $\boldsymbol{x}$ direction, $l = 270\,\mu\mathrm{m}$, $p = 1.116$ mm, $d_x = d_y = 508\,\mu\mathrm{m}$, $d_z = 15\,\mu\mathrm{m}$, $R_x = 25\,\mu\mathrm{m}$, $R_z = 25\,\mu\mathrm{m}$, $(\epsilon_{rf}, \sigma_f) = (1, 5.8 \cdot 10^7\ \mathrm{S/m})$, $(\epsilon_{rm}, \sigma_m) = (3.4, 50 \cdot 10^{-9}\ \mathrm{S/m})$.

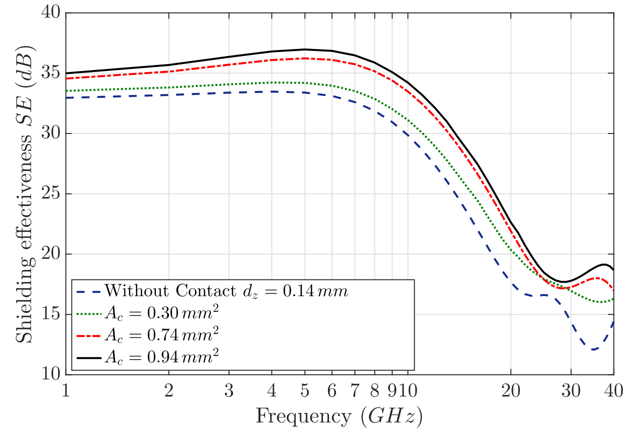

Fig. 6. SE of an infinite sheet (woven fiber composites of Fig. 2): variation according to contact surface area A_c. Configuration: electric field polarized in the $\boldsymbol{x}$ direction, fiber concentration ratio $f_v = 17.5\%$, $l = 3\,\mathrm{mm}$, $p = 4\,\mathrm{mm}$, $d_x = d_y = 1.22\,\mathrm{mm}$, $R_x = 625\,\mu\mathrm{m}$, $R_z = 250\,\mu\mathrm{m}$, $(\epsilon_{rf}, \sigma_f) = (1, 10^3\ \mathrm{S/m})$, $(\epsilon_{rm}, \sigma_m) = (5, 0\,\mathrm{S/m})$.

that the finite element model studied here is suitable to simulate this geometry. Fig. 5 shows the obtained results. Over the frequency range of 1–100 GHz, the obtained results match those of conducted study, with a few differences that can be caused by quality of the mesh when approaching the higher frequencies.

The manufacturing process of woven composite materials establishes an elastic contact between the fibers [24]. This results in a variation of the electrical resistivity of the contact area according to [25]. This effect is integrated into the FEM model by adding thin layers of conductive materials between the fibers to represent the electric contact. By varying the surface of these layers the effect of the contact ratio can be considered. Fig. 6 shows that vertical contact between fibers reinforces SE by introducing closed paths for the induced currents to circulate. Because the contact ratio between fibers is dependent on the weaving and resin molding process (as well as other mechanical and thermal influencing factors), it varies from one weave to another inside the same fabric. Its estimation should therefore be based on experimental data. Moreover, the study of contacts can be more complicated due to asperities [26]. In this case the contact area can indeed be obtained by numerical fitting.

B. Homogenization Results

The proposed homogenization technique is carried out for the woven fiber composite of Fig. 2. The computed effective properties as well as the SE and reflection coefficient estimates computed using this technique are plotted in Fig. 7 (in this case study, the weighing coefficients are: $a = |R_{\mathrm{FEM}}(f)|$ and $b = |\mathrm{SE}_{\mathrm{FEM}}(f)|$). Results demonstrate a good agreement between numerical calculations and the homogenized medium, with an estimated error of approximately 5% in this case. Additionally, multiple studies consisting in carrying out this homogenization method over different woven fiber composites (parametric studies) demonstrate its tendency to capture with good accuracy their frequency behavior by computing a corresponding effective medium (Fig. 8). Here the used weighing coefficients are: $a = |R_{\mathrm{FEM}}(f)|$ and $b = 1$. The obtained results show that by simulating a unit cell of a heterogeneous material, we are able to define an equivalent homogeneous medium at the macroscopic scale. This is useful when conducting large-scale simulations of realistic structures for EM compatibility applications.

IV. Equivalent Two-Layer Model

In order to evaluate the woven fiber model presented in the previous section, a comparison with analytical formulas was performed to check their ability to predict the frequency response. To this end a simplified model of the woven composite is proposed as a stepping stone to apply analytical mixing rules. The similarities between the two materials are detailed at first, the simulation results are then analyzed.

From an electrical viewpoint, the woven fiber microstructure is essentially made of two sets of fibers. A set of unidirectional fibers laying perpendicularly to the incident electric field, and a

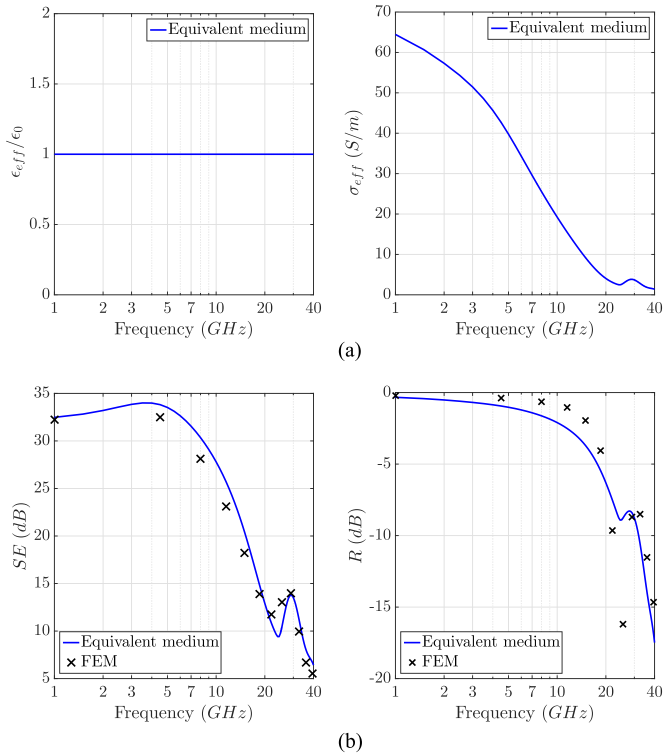

Fig. 7. FEM and homogenization results of a woven fiber composite. Configuration: electric field polarized in the $\boldsymbol{x}$ direction, fiber concentration ratio $f_v = 16.9\%$, $l = 3\,\text{mm}$, $p = 4\,\text{mm}$, $d_x = d_y = 1.22\,\text{mm}$, $d_z = 140\,\mu\text{m}$, $R_x = R_z = 380\,\mu\text{m}$, $(\epsilon_{rf}, \sigma_f) = (5, 10^3\ \text{S/m})$, $(\epsilon_{rm}, \sigma_m) = (5, 0\ \text{S/m})$. (a) Effective properties (permittivity and conductivity). (b) Shielding and reflection coefficients.

second set of the same fibers which are parallel to the electric field. Moreover, the material being diagonally anisotropic, the effective permittivity takes the form of a diagonal dyadic. Additionally, the symmetry in the (xy) plane gives $\epsilon_x^* = \epsilon_y^*$. This is also the case for a composite material containing two layers of unidirectional fibers set in opposite directions (see Fig. 9).

The relevance of this analogy is verified by computing the SE for both materials using FEM calculations. The calculation conditions are similar to those in Fig. 3 with periodic boundary conditions, except that the composite sheet is replaced by the unit cell of the simple material. The results show good agreement between the shielding properties of the two materials, notably when comparing woven fibers with contact to the two layered laminate composite (see Fig. 10). The variation patterns are equally conserved. Thus, the applicability of analytical EM homogenization rules can be studied by reducing the woven structure to that of a two layered laminate with infinitely long cylindrical inclusions.

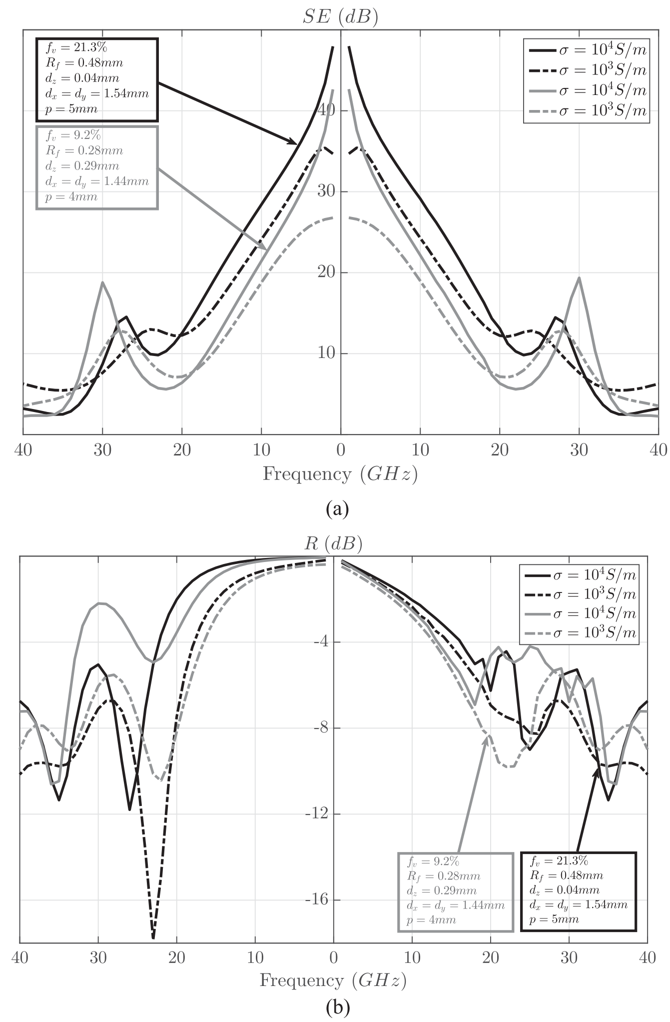

Fig. 8. FEM (left) and homogenization (right) results for multiple configurations based on Fig. 2. The electric field is polarized in the $\boldsymbol{x}$ direction. (a) SE of different woven fiber composites. (b) Reflection coefficient of different woven fiber composites.

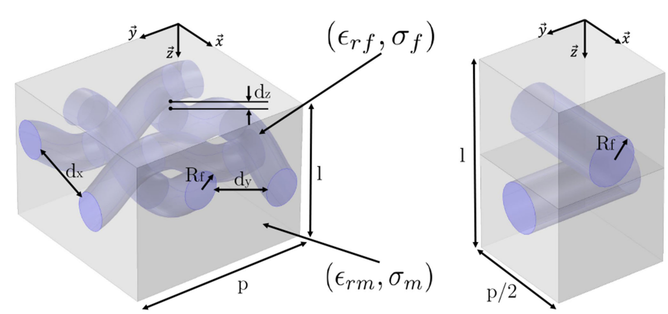

Fig. 9. Woven fiber composite versus two layered unidirectional fiber laminate composite.

V. Analytical Homogenization Methods

Since the woven fabric material problem can be reduced to that of a two layered composite containing cylindrical inclusions, the corresponding homogenization problem can be solved by using analytical mixing rules.

A. Inclusion-Based Problems

In order to choose appropriate formulas for computing the effective properties, the homogenization problem is solved using an inclusion-based method [27], [28] as represented in Fig. 11.

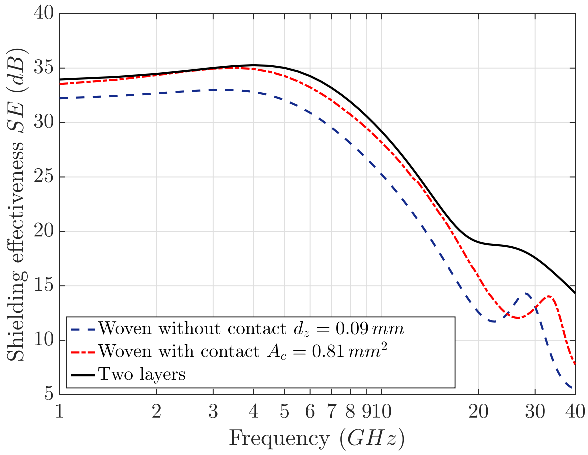

Fig. 10. SE of an infinite sheet (woven fiber composites versus two layered composite of Fig. 9) with respect to frequency. Configuration: electric field polarized in the $\boldsymbol{x}$ direction, fiber concentration ratio $f_v = 16.9\%$, $l = 3$ mm, $p = 4$ mm, $d_x = d_y = 1.22$ mm, $d_z = 140\,\mu$m, $R_f = 380\,\mu$m, $(\epsilon_{rf}, \sigma_f) = (5, 10^3$ S/m), $(\epsilon_{rm}, \sigma_m) = (5, 0$ S/m).

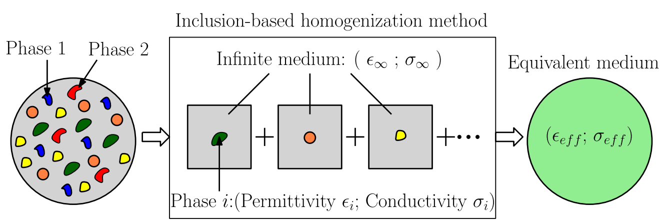

Fig. 11. Inclusion-based homogenization method.

Each ellipsoidal phase of the heterogeneous material is located in an unbounded region called infinite medium, which is defined by its electrical properties ($\underline{\underline{\epsilon}}^*_\infty = \underline{\underline{\epsilon}}_\infty - \jmath\frac{\underline{\underline{\sigma}}_\infty}{\omega}$) for a harmonic excitation. The interaction between the external electric field and the phase is then calculated by introducing the depolarization tensor of the phase $\underline{\underline{N}}_i$ as well as its electrical permittivity $\underline{\underline{\epsilon}}^*_i$. The average of these individual responses gives the effective permittivity $\underline{\underline{\epsilon}}^*_{eff}$ [27]

$$\underline{\underline{\epsilon}}^*_{eff} = \left\langle \underline{\underline{\epsilon}}^*_i \cdot \left(\underline{\underline{I}} + \underline{\underline{N}}_i \cdot \underline{\underline{\epsilon}}^{*-1}_\infty \cdot \left(\underline{\underline{\epsilon}}^*_i - \underline{\underline{\epsilon}}^*_\infty\right)\right)^{-1} \right\rangle \left\langle \left(\underline{\underline{I}} + \underline{\underline{N}}_i \cdot \underline{\underline{\epsilon}}^{*-1}_\infty \cdot \left(\underline{\underline{\epsilon}}^*_i - \underline{\underline{\epsilon}}^*_\infty\right)\right)^{-1} \right\rangle^{-1} \quad (9)$$

where the operator $\langle . \rangle$ denotes a volumetric average operation and $\underline{\underline{I}}$ is the second-order identity tensor.

B. Analytical Homogenization of Two-Layer Composites

When applying (9) to the laminate composite, the homogenization is divided into three steps as shown in Fig. 12:

1) Layer of fibers parallel to the incident electric field: in this case, the depolarization tensor $\underline{\underline{N_a}}$ is written as

$$\underline{\underline{N_a}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix}_{(\boldsymbol{u_x}, \boldsymbol{u_y}, \boldsymbol{u_z})} . \quad (10)$$

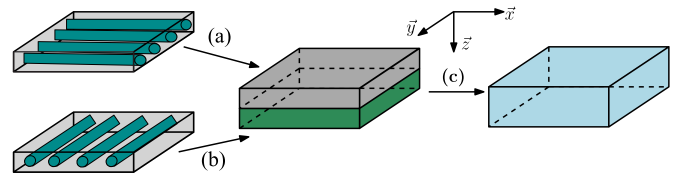

Fig. 12. Homogenization process of a two layered composite.

The effective properties can thus be obtained using the upper bound of Wiener's formula [29].

2) Layer of fibers perpendicular to the incident electric field: the depolarization tensor $\underline{\underline{N_b}}$ is expressed as

$$\underline{\underline{N_b}} = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix}_{(\boldsymbol{u_x}, \boldsymbol{u_y}, \boldsymbol{u_z})} . \quad (11)$$

In this case, to take into account the interaction between the inclusions and incident wave at relatively low-volume fractions, the infinite medium is chosen as follows [6], [30]:

$$\epsilon^*_\infty = \left(\left(\epsilon_m + \jmath\frac{\sigma_m}{\omega}\right) + \left(\epsilon_f + \jmath\frac{\sigma_f}{\omega}\right) \cdot \left(\frac{2R_f}{\lambda}\right)^2\right) \cdot \underline{\underline{I}} \quad (12)$$

with (ϵ_m, σ_m) and (ϵ_f, σ_f) the permittivities of the dielectric matrix and the conductive fibers respectively. λ is the wavelength of the incident wave. The effective property in the direction u is then reduced to that of the dynamic homogenization model [30]

$$\epsilon^u_b = \frac{\epsilon^*_f \dfrac{f_v}{\epsilon^*_\infty + N^u_b\left(\epsilon^*_f - \epsilon^*_\infty\right)} + \epsilon^*_m \dfrac{(1 - f_v)}{\epsilon^*_\infty + N^u_b\left(\epsilon^*_m - \epsilon^*_\infty\right)}}{\dfrac{f_v}{\epsilon^*_\infty + N^u_b\left(\epsilon^*_f - \epsilon^*_\infty\right)} + \dfrac{(1 - f_v)}{\epsilon^*_\infty + N^u_b\left(\epsilon^*_m - \epsilon^*_\infty\right)}} . \quad (13)$$

3) Two consecutive layers of homogeneous sheets: the depolarization tensor is

$$\underline{\underline{N_c}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}_{(\boldsymbol{u_x}, \boldsymbol{u_y}, \boldsymbol{u_z})} . \quad (14)$$

The mixing rule for the final step also reduces to Wiener's upper bound formula.

The SE of the equivalent medium is obtained using the formula detailed in equations (2)–(6) and compared to that simulated using FEM (curves labeled "Homogenization" and "FEM:1 fiber" of Fig. 13). The estimate provided by the analytical homogenization is close to that of the simulated heterogeneous material for the lower frequencies. However, as the frequency increases, results start to diverge. The analytical estimates are no longer able to accurately predict the response of the composite.

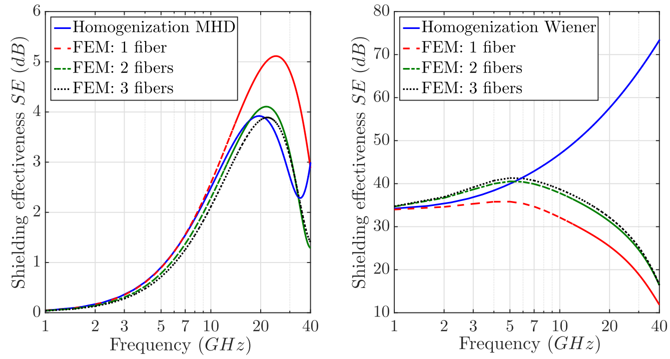

Fig. 13. Evolution of the SE as a function of the number of stacked fibers in the z direction: fibers perpendicular (left) and parallel (right) to the incident electric field. Configuration: fiber concentration ratio $f_v = 16.9\%$, $l = 3\,\text{mm}$, $p = 4\,\text{mm}$, $(\epsilon_{rf}, \sigma_f) = (5, 10^3\ \text{S/m})$, $(\epsilon_{rm}, \sigma_m) = (5, 0\ \text{S/m})$.

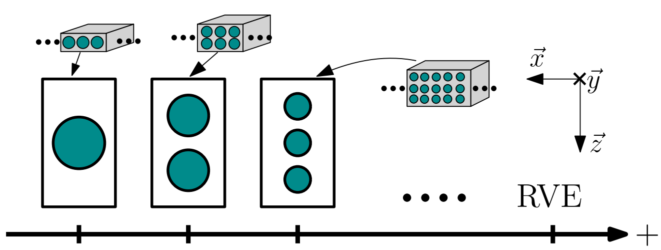

Fig. 14. Stacking fibers in the z direction of the composite in order to form a representative volume element.

C. Representative Volume Element (RVE) and Dynamic Effects

The discrepancies between the numerical and analytical results can be explained. Indeed, analytical mixing rules were established under certain specific assumptions concerning the material and EM solicitation. One of these hypotheses is the existence of a RVE [31] which contains a large number of micro heterogeneities. This condition is not satisfied along the z dimension where only two fibers can be found.

To verify the validity of this assumption, each of the two layers of the composite is studied separately. One made of fibers perpendicular to the electric field, and the other containing fibers that are parallel to the electric field. For each laminate, the study consists in consecutively stacking additional layers of inclusion as shown in Fig. 14. At each step the fibers volume fraction remains unchanged (this is done by varying the radius, all other dimensions remaining the same). The fibers occupy a total volume that is proportional to the square of their radius making it not possible to add a fourth layer given the sheet dimensions.

The simulated results (Fig. 13) show the evolution of the material response when more fibers are stacked. In fact, the added layers bring the composite closer to satisfying the RVE condition, thus getting the response to converge towards that predicted by the analytical formulas. As for the one layer configuration, it is particularly influenced by the phenomena that occur at higher frequencies, e.g. the proximity and skin effect, which can have a paramount importance on its EM response. That is to say, assuming a quasi-static excitation and the presence of a RVE, mixing rules will predict a SE increase with frequency. However, as frequency increases (and keeping the RVE condition), it is shown in [6] that the skin effect will decrease the SE. Moreover, if now we consider only one layer of fiber in the material, the total induced current will drop even more leading to a decreasing SE with respect to frequency. This explains the trends represented in the figure. This also implies that currently the classical mixing rules cannot accurately predict the effective properties of a two-layer composite, which means that they are equally not suitable for woven fiber composites. In summary, analytical mixing rules derived from the general homogenization method described by inclusion-based problems is not applicable in this case. This is because the material does not satisfy the RVE condition. Though the homogenization technique carried out in this paper can be computationally heavy, it has shown to give an accurate description of the woven fabric's EM behavior.

VI. Conclusion

In this paper, the EM response of conductive woven fiber composite materials is studied for applications related to EM compatibility. For a plane wave configuration, FEM computations demonstrate stable levels of SE for the lower frequency range. Beyond a certain cutoff frequency, results show a drop in the estimated effectiveness. Large scale modeling of composite materials is rendered feasible using EM homogenization. To this end, a homogenization technique based on finite element simulations on a periodic cell of the microstructure combined with numerical optimization is carried out. Effective electrical properties thereby obtained show accurate description of the woven fiber composites behavior with a mean error margin lower than 12% over the studied frequency band. Analytical mixing rules are then proved inadequate for estimating an EM effective medium for woven fabric composites at higher frequencies. This is caused by the structure of the composite which, at the mesoscopic scale, contains one layer of fibers. This is not sufficient for the application of analytical mixing rules because they require a RVE. While the case of normal incidence is interesting to globally describe shielding levels, an oblique incidence study, which remains to be carried out, would provide a more realistic base for shielding enclosure applications.

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